## EDITOR'S PREFACE

This translation of the *Grundzüge der Theoretischen Logik* of Hilbert and Ackermann has been made from the second German edition, which was published in 1938 and has since enjoyed the status, assuredly well-merited, of a classic text in the field of mathematical logic. Those who have cooperated in the translation have sought both to give an exact English rendering of the sense and intent of the original text and also, so far as possible in a different language, to reproduce something of its manner and style. It has nevertheless been judged necessary to depart in some respects from the letter of the German text at places where, in the light of the general advance in precision of logical terminology since the text was written, its formulations now seem ambiguous or otherwise imperfect, and especially at places where technical criticism of the text itself has shown it to be in error. It is one of the purposes of this preface to call the reader's attention to these changes, for which the editor must assume responsibility.

In addition to the correction of a few typographical errors, a number of minor changes have been made throughout the text in correction of the recurrent carelessness of the authors in maintaining a strict distinction (according to present standards) between expressions and the objects designated by them; but changes of this kind have been in every case very slight and have been made only when they seemed necessary in order to avoid serious ambiguity or terminological inconsistency. The reader will observe in this connection that the translators, like the authors, have adopted the customary convention, although without explicit statement thereof, that the symbols and formulas of a logical calculus (object language), when mentioned, or spoken about, in the metalinguistic discussion, may be used autonymously, *i.e.* as names for themselves. And indeed this convention is extended to cover mention of the special symbols (such as the German letters) of the metalanguage as well.

The first of the more serious errors which critical examination of the German text has revealed is one of omission: namely, the authors failed to include an essential condition relative to bound variables in their statement of the rule of substitution for predicate variables [Rule $\alpha 3$), pp. 69f. of the translation]. This error was pointed out by Professor Alonzo Church in his monograph of 1944 (*Introduction to Mathematical Logic*) and has been corrected by the insertion of a clause embodying the amendment which he suggested.

A second important difficulty concerns the introduction to the argument of Gödel's completeness proof for the predicate calculus. The inadequacy of the authors' formulation at this point was shown by Professor W. V. Quine in his review of the second German edition at the time of its original appearance.[1] The difficulty is the following: The authors state, in effect, that since they have proved that for every formula of the predicate calculus a formula in Skolem normal form can be found such that either both formulas are provable or both are not provable, they are free to restrict themselves, in carrying out the desired completeness proof, to showing that all universally valid formulas in Skolem normal form are provable. But the conclusion which they draw here would seem to be a *non-sequitur*, since they have nowhere shown that the formula in Skolem normal form corresponding to a given universally valid formula must also be universally valid. Therefore, in the text of the translation a brief paragraph (*viz.* the final paragraph of § 8) has been inserted indicating a method of proof by which the reader may fill in this gap in the argument of the completeness proof, and the inadequate statement at the beginning of Gödel's argument has been amended accordingly.

The third and most considerable departure in the translation from the original text again concerns the development of Gödel's completeness proof, but this time consists in replacing the erroneous proof of part (A) of Gödel's argument by a corrected proof (pp. 98f). In connection with this corrected proof, the editor is again indebted for suggestions to the writings of Professor

[1] The Journal of Symbolic Logic, Vol. 3 (1938), p. 84.

Church, specifically to his proof of the same proposition in his monograph of 1944 (pp. 75f) as subsequently revised in the review of the monograph in the Journal of Symbolic Logic [Vol. 10 (1945) p. 20 ].

Finally, the editor has added a small number of notes, some of which appear at the back of the book, in supplementation of the translated text. The purpose of the appended notes is to explain various terminological usages in the translation, particularly with a view to helping the student to whom the subject of logic is new and who may wish to correlate his study of this work with his readings in other texts.

In concluding this preface, the editor wishes on behalf of the translators and on his own behalf to thank Mr. J. Schwartz for his kindness in making available for purposes of comparison the manuscript of an independent translation of the same work which he had prepared. We are gratefully indebted to Mr. Schwartz for many valuable suggestions and improvements in expression.

New Brunswick, N. J.

**Robert E. Luce**

## PREFACE TO THE FIRST (GERMAN) EDITION

The present work treats mathematical logic (also called symbolic logic, logistic, or the algebra of logic) in a form that I have developed and used in my university lectures on the fundamental principles of mathematics (*Principles of Mathematics*, Winter Session 1917-18; *Logical Calculus*, Winter Session 1920; *Foundations of Mathematics*, Winter Session 1921-22). In the preparation of these lectures I have had considerable aid and advice from my colleague P. Bernays; he has also written the lectures up most carefully. The material thus obtained has been used and supplemented, to give the present arrangement and definitive presentation of the entire subject matter, by W. Ackermann, a student of mine who has since distinguished himself by important papers of his own on the foundations of mathematics.

This book is intended to serve at the same time as an introduction to and preparation for a further book, which P. Bernays and I plan to publish soon and which treats the foundations of mathematics by the method I have expounded—again with the active cooperation of P. Bernays—in a series of articles (*Neubegründung der Mathematik*, Abhandlungen des mathematischen Seminars der Hamburgischen Universität, Vol. 1, p. 157 (1922); *Die logischen Grundlagen der Mathematik*, Math. Ann. Vol. 88, p. 151 (1922); *Über das Unendliche*, Math. Ann. Vol. 95 p. 161 (1925)).

Göttingen, January 16, 1928

**Hilbert**

## PREFACE TO THE SECOND (GERMAN) EDITION

The second edition of the present work retains the arrangement and form of the first edition throughout. However, the progress which has been made in the subject since the appearance of the First Edition has made it necessary to go over the book in detail and to include various improvements and additions. This has been done without going beyond the limits set for the book.

The first and second chapters are essentially unchanged, except that recent investigations of the axiomatic basis of the sentential calculus have been briefly taken into account in the first chapter. A more extensive presentation of the class calculus in the second chapter, which might have been desirable for its own sake, was not given, since this calculus occupies, after all, an isolated position in the overall arrangement of the book. In the third chapter, the formulation of the rules of inference for the predicate calculus, which was not sufficiently precise, has been improved. Proofs of the independence and completeness of the system of axioms used there have been newly added, and the section on the decision problem has been supplemented by incorporating recent results. It was possible to shorten the fourth chapter inasmuch as it was no longer necessary to go into Whitehead and Russell's ramified theory of types, since it seems to have been generally abandoned. On the other hand, the structure of the predicate calculus of second order and of the calculus of order $\omega$ has been considerably improved and rounded out.

The terminology has been adapted to that of the *Grundlagen der Mathematik* by Hilbert and Bernays. For example, the term "functional calculus" has been everywhere replaced by "predicate calculus." Following general logical usage, the expressions "logical sum" and "logical product" have been changed throughout to "conjunction" and "disjunction."

For much helpful advice I am especially indebted to Professor P. Bernays of Zürich, who also read the galley proofs. I am also grateful for various suggestions to Mr. G. Gentzen of Göttingen, who also examined the manuscript, as well as to Messrs. Arnold Schmidt of Marburg and H. Scholz of Münster. To all of them I offer my most cordial thanks.

Burgsteinfurt, November, 1937

**W. Ackermann**