Preface

These are lecture notes for a one-semester graduate class in mechanics given in 2006 and 2007 to first-year Ph.D. students and advanced undergraduates at the Courant Institute of Mathematical Sciences. The idea was to present an introduction to the methods of classical, statistical, and quantum mechanics to Ph.D. students who may be at the beginning of their research in applied mathematics, mathematical physics, or applied probability. No previous knowledge of physics was required.

A traditional in-depth study of mechanics in physics or engineering is a matter of years and cannot be compressed into a one-semester course. However, the most time-consuming task in learning mechanics is the idealized modeling of a physical problem, which requires working through many examples in order to build up the right kind of intuition. On the other hand, a beginning Ph.D. student in mathematics may be willing to take an existing idealized model on trust, or to work through its derivation for the special thesis problem at hand. Nevertheless, he or she might still benefit from a knowledge and appreciation of the basic methods and concepts in mechanics. To provide this kind of overview was the aim of the course.

All of the material in this course is standard and readily available in textbooks and review articles (some references are suggested in the bibliography), though I believe there is no one place were all the topics could be found. To the best of my ability, I have chosen topics that stress fundamental mathematical methods, because these methods provide connections across seemingly unrelated material. An example of this is Hamilton–Jacobi theory, which appears in the calculus of variations, in Fermat's principle of classical mechanics, and in the geometric theory of dispersive wavetrains. Another example is path-integral methods, which appear in quantum mechanics and in statistical mechanics. The latter application points to the far-reaching modern generalizations of these methods in diffusion theory and probability. That was the appropriate place to end this course.

It has been a lot of fun teaching this class, and I would like to thank all the students who participated, especially for those lively questions that kept us on our toes. Finally, I would like to thank Esteban Tabak and Jalal Shatah for their encouragement to write these lecture notes.