

Preface

In fall 2006, I taught a course on semiclassical pseudodifferential operators at the Courant Institute of New York University. In the semester that followed, I taught a course called “Wigner measure and semiclassical limits of nonlinear Schrödinger equations” there. The main purpose of the spring course was to apply the theory of semiclassical pseudodifferential operators to the study of various high-frequency limits of equations from quantum mechanics. In particular, we presented detailed explanations of Wigner measure and then tried to summarize some recent progress on how to use this tool to study various problems on semiclassical limits of nonlinear Schrödinger equations.

Because there are already some very nice books on semiclassical pseudodifferential operators, such as those by D. Robert [84] and A. Martinez [72], I mainly present the material from my spring course in 2007.

This book is organized into seven chapters. Chapter 1 deals with the classical WKB method. Chapter 2 deals with the theory of Wigner measure. Chapter 3 deals with the semiclassical limit from one-dimensional Schrödinger-Poisson to Vlasov-Poisson equations. Chapter 4 deals with the semiclassical limits of multidimensional Schrödinger-Poisson equations. Chapter 5 deals with semiclassical limits of the cubic Schrödinger equation in an exterior domain. Chapter 6 covers semiclassical limits of the coupled nonlinear Schrödinger system. In the last chapter, we shall digress a little bit from the main topic and talk about the applications of Wigner measure to the study of high-frequency limits of Helmholtz equations. Finally, we have four appendices concerning the global existence of smooth solutions to various Schrödinger-type equations.

I would like to take this opportunity to thank Professor Charles M. Newman for inviting me to visit Courant Institute from the fall of 2005 to the spring of 2007. It is my great pleasure to thank Prof. Fanghua Lin and Prof. Jalal Shatah for encouraging me to write down these notes. It is also my great pleasure to thank all of my collaborators on this topic: Profs. Fanghua Lin, Taichia Lin, Norbert J. Mauser, Xueping Wang, and Yuxi Zheng. I should also thank my class at the Courant Institute: Diogo Arsénio, Hantaek Baik, Jose Diaz-Alban, David Gérard-Varet, Pierre Germain, Nam Le, Ian Tice, Yong Yu, and Zhifei Zhang, with special thanks to Zhifei Zhang for reading the draft version of this book. Finally, it was a pleasure to collaborate with Paul Monsour in his beautiful editing work.

Part of Chapter 2 was also presented by the author in the summer school at Fudan University and Wuhan University, China, in the summer of 2007. I would

like to thank Profs. Jiaying Hong and Hua Chen for their kind invitation and constant support. I would also like to acknowledge the support from the National Science Foundation of China under grants 10131050 and 10276036, the National “973” Project, and the Chinese Academy of Sciences through an Innovation Grant.