Preface

Microelectromechanical systems (MEMS) and *nanoelectromechanical systems* (NEMS), which combine electronics with miniature-size mechanical devices, are essential components of the modern technology that is currently driving telecommunications, commercial systems, biomedical engineering, and space exploration. These are only a few of the vast number of applications that lie at the roots of microsystem technology. Over the years, and in order to provide accurate, controlled, and stable locomotion for such microdevices, researchers have proposed a variety of modes, based upon thermal, biological, or electrostatic forces. It is the mathematical model describing the method of "electrostatic actuation" that we shall address in this monograph. The process is based on an electrostatically controlled tunable capacitor that is widely used in microresonators, optical microswitches, chemical sensors, micromirrors, accelerometers for airbag development of automobiles, micropumps for inkjet printer heads, microvalves, shuffle motors, micro- and nanotweezers, among many other devices.

There now exist many variations in electrostatic actuation technology. They are all, however, based on a simple physical principle relating

- the elastic deformation which—by elementary plate theory—depends on the Laplacian of the deformation variable (to account for stretching), and on its bi-Laplacian (for bending),
- the electrostatic force which—by the classical Coulomb law—is proportional to the inverse square of the distance between the two charged plates, itself a function of the deformation variable.

Unfortunately, models for electrostatically actuated microplates that account for moderately large deflections and which do not assume that each material point moves vertically over its reference position are quite complicated and not yet amenable to rigorous mathematical analysis. In this book we deal with much simplified models that still lead to very interesting second- and fourth-order nonlinear elliptic equations (in the stationary case) and to nonlinear parabolic equations (in the dynamic case). The nonlinearity is of an inverse square type, which until recently has not received much attention as a mathematical problem. It was therefore rewarding to see, besides the above practical considerations, that the model is actually a very rich source of interesting mathematical phenomena. Numerics and formal asymptotic analysis give lots of information and point to many conjectures, but even in the simplest idealized versions of electrostatic MEMS, one essentially needs the full available arsenal of modern nonlinear analysis and PDE techniques "to do" the required mathematics. Indeed, while nonlinear eigenvalue problems—where the simplified MEMS models seem to fit—are a well-developed field of PDEs, the type of nonlinearity that appears here helps to shed a new light on the class of singular supercritical problems and their specific challenges. Furthermore, these fourth-order models for MEMS are also amplifying the need for a better and deeper understanding of equations involving the biharmonic Laplacian, which remain quite elusive in spite of recent advances. The dynamic case presents its own challenges, which have only been tackled in the parabolic setting so far, while its second-order wavelike counterpart is still completely open to mathematical inquiry.

It is therefore our objective to present in this text a rigorous mathematical analysis for various phenomena related to some of the simplest proposed models, many of which were observed either numerically or via ODE methods in the radially symmetric case. Our goal is to try to contribute to the practical needs of engineers and manufacturers, while satisfying at the same time the intellectual curiosity and the quest for rigor of research mathematicians. A case in point are the estimates on "pull-in voltages" and "pull-in distances" that depend on the size and geometry of the domain and on the permittivity profile of the membrane, which have obvious practical considerations. On the other hand, pull-in voltage estimates that also depend on the dimension of the ambient space may only be of interest to mathematicians whenever one goes beyond two-dimensional space. A similar dependence occurs for the refined properties of steady states—such as regularity, stability, uniqueness, multiplicity, energy estimates, and comparison results. The same complexity carries to the dynamic case where issues related to the "quenching profile"—in finite or infinite time—or to global convergence towards a stable steady state, present many interesting mathematical challenges.

From the pedagogical point of view, this monograph is definitely meant for those already familiar with the modern theory of partial differential equations. It may, however, offer an unusual opportunity as an advanced graduate text: a motivational introduction to the most recent methods of nonlinear analysis and PDEs through the analysis of a set of equations that have enormous practical significance. Indeed, as mentioned above, the analysis of this most simple idealized version of electrostatic MEMS seems to require the "kitchen sink" of modern tools in PDEs: the notions of weak, sub- and supersolutions, bifurcation diagrams and their connection to Morse theory, energy estimates via Sobolev spaces and Moser's iteration, compactness via blowup phenomena and nonlinear Liouville theorems, uniqueness via monotonicity formulae and Pohozaev identities, as well as profile analysis via maximum principles and moving plane methods. None of these required tools is detailed here (an impossible task), but our hope is that their efficacy is displayed enough so that this book can serve as a motivational reference for learning and practicing these powerful tools of mathematical analysis.

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