Introduction

The present course will deal with electromagnetic theory from a mathematical point of view. That does not mean that this theory is used just as a playground for mathematical methods or simply as a reservoir of problems to which mathematical theorems can be applied. Electromagnetic theory will be taken for what it is, a theory of physics. Still the attitude in dealing with this theory may be different from the one naturally taken by a physicist or an electro-engineer.

Take the question: what is electricity? In one book this is said: "If a stick of sealing-wax is rubbed with cat's fur, both bodies are put into a peculiar condition in which light bodies in their neighborhood are set in motion." In another book it is taken for granted that there is a socket in the baseboard in your room and electricity is described by the peculiar happenings that can take place at the end of a pair of wires plugged into the socket. If, in particular, one puts an electric razor at the end of the pair of wires and observes that it runs, one will be led to the definition: what makes the razor run, that is electricity.

We shall not adopt such a "technological" definition of electricity, nor the "romantic" sealing-wax and cat's fur definition. We shall characterize in rather general terms electricity by the specific manner in which electric forces act on objects and depend on the agent that produces them. We shall show that in this manner of action and dependence electric forces differ in a purely formal way from gravitational and elastic forces. Thus our definition of electricity might be called an axiomatic one, although this term has some unwanted connotations. In explaining the phenomena of electricity and magnetism, physicists frequently follow the way in which an understanding of these phenomena has developed historically, or they present the simplest realistic experiments that exhibit these phenomena most pointedly. We shall assume the present-day understanding of the theory (or more precisely, the understanding reached after the advent of the special theory of relativity in 1905), and we shall exemplify the basic laws, not by the simplest possible realistic experiments, but by simple fictitious experiments that represent these laws most directly without technological encumbrances.

This approach is not meant to be "the" right approach—far from it. It is meant to lead to one out of several aspects of the theory of electricity and magnetism, all of which together contribute to an understanding of these phenomena.

The title of the course involves the notion of methods. To be sure, there are mathematical methods for solving mathematical problems that express physical questions. A "solution" of such a problem may lead to a numerical evaluation, but frequently it will help us to understand the phenomena and lead to overall expectations. We shall interpret the term "method" in still a wider sense. Specifically, this term is also meant to cover mathematical methods of proper formulation of physical laws; thus it includes the vector and tensor analysis in Euclidean and Minkowskian space; but more generally it is to cover mathematical results that show what kind of differential equations are suitable as expressions of physical laws.

The term "physical laws" here refers not only to the basic ones, but also to laws derived by various simplifying assumptions. We shall frequently discuss the question how such simplifications can be justified, but we shall rarely do this rigorously. In fact, we shall not at all indulge in the sport of being rigorous in the strict sense in which this notion is understood in mathematics.