

## Preface

These notes have been used for several years, by various instructors, in a one-year course sequence, Linear Algebra I and Linear Algebra II, offered in the Mathematics Department at New York University's Courant Institute. These courses were developed with two audiences in mind:

- As Senior Honors courses for undergraduate mathematics majors seeking a stronger background in linear algebra.
- As part of a one-year course sequence for students in the Courant Master's Degree Program in Mathematics. This large and diverse audience includes:
  - (i) ambitious undergraduate math majors seeking a deeper understanding of linear algebra,
  - (ii) graduate students from other quantitative disciplines, particularly computer science and economics, intent on building a stronger math background,
  - (iii) mathematically proficient students with undergraduate backgrounds in mathematics, the sciences, or engineering, who after working for a few years have developed a strong interest in Ph.D.-level studies in pure or applied mathematics.

A significant number of students in the last category have been accepted with advanced standing into the regular Courant Ph.D. program after completing their one-year master's program and passing written qualifying exams.

Accordingly, these notes are not a comprehensive introduction to linear algebra. Students were presumed to have had a prior course in linear algebra, a basic understanding of matrix algebra, and some proficiency with mathematical proofs. However, proofs are emphasized in this course sequence as a means of building students' skills at analyzing problems for themselves, anticipating the day when they will encounter questions that cannot be resolved by a Google search.

The overall emphasis in these notes is on understanding the structure of linear operators as the key to solving problems in which they arise.

Although these notes provide a self-contained tour of linear algebra, in practice they were often supplemented by two (optional) standard texts that

provided alternative views of many topics as well as more extensive problem sets, and are well-aligned with the notes. These were:

1. *Linear Algebra* by Seymour Lipschutz, 2<sup>nd</sup> edition, Schaum's Outline Series, McGraw-Hill, 1991.
2. *Linear Algebra* by Stephen Friedberg, Arnold Insel, and Lawrence Spence, 4<sup>th</sup> edition, Prentice-Hall, 2003.

Students found the Lipschutz text particularly helpful for its clarity, well-chosen worked examples, extensive selection of exercises, and detailed discussion of proofs. The first four chapters also provide a concise and self-contained review of the topics that are prerequisites to this course: basic matrix algebra, row operations, and solution of systems of linear equations.

### **Organization of the Text**

The handling of exercises is somewhat unconventional. Many are placed within the main text, as the topics they address first occur. Each chapter ends with an extensive set of section-by-section Additional Exercises. Some recap the main topics of each section; others are longer and intended to be more challenging; each begins with a block of true/false questions, which students often find more challenging than you might expect.

There are a few unconventional notations, explained as they are introduced:

- We often write  $|V|$  for the dimension  $\dim(V)$ , and  $R(T)$ ,  $K(T)$  for the range and kernel of  $T$ , respectively, when this is convenient.
- A special symbol  $\mathbf{1}$  is used to distinguish the constant function (or polynomial) everywhere equal to 1, from the scalar 1.

The symbol  $\mathbb{K}$  indicates a generic ground field.

### **Acknowledgments**

The present volume, *Linear Algebra I*, covers the first semester of the NYU course sequence Graduate Linear Algebra I–II. The companion volume, *Linear Algebra II*, used in the second semester, is currently being prepared for publication in the Courant Lecture Notes series.

These Notes were developed over several years, as handwritten lecture notes for Professor Greenleaf's courses. They were reproduced and distributed to students along with detailed problem sets (many with full solutions).

The typeset  $\LaTeX$  version of these Notes might never have seen the light of day without the enthusiastic and tireless efforts of the second author, Dr. Sophie Marques, who came to Courant as a postdoc in 2014 and was assigned to teach the Linear Algebra I–II sequence. She quickly converted the existing notes into serviceable  $\LaTeX$  code, and during the next two years actively helped reshape the final text based on her own experience teaching these courses. The final version owes a lot to those efforts, and her talent as a teacher.