

# Preface

*Tell me, and I forget. Teach me, and I may remember. Involve me, and I learn.*

—Ancient Proverb

Suppose you are scheduled to visit a college math classroom and find feather boas, hula hoops, cards, balls, jacks, or a large stack of cookies. Perhaps you think the class is somewhere else, and this classroom is set up for young children. Thinking back on your own preschool days, you know that using tactile activities for learning is commonly accepted and well documented in early education; the learning theories of Froebel, Montessori, Piaget, and Erikson all indicate the importance of hands-on learning. While you may have heard of experiential learning at the college level, that term typically refers to internships or lab classes, and you are fairly certain it doesn't apply to mathematics. So you check the sign on the door, and sure enough, this is the correct math class, taught by any one of the contributors to this book, including the two of us.

The value of active learning, which includes tactile learning, is well documented in the literature. The benefits of these active learning approaches include increased accessibility, deeper understanding, and a way to level the playing field for students with diverse backgrounds and levels of mathematical preparation [3, 5, 7, 8]. In addition to the strong evidence in the literature, both of us have had the joy of observing our students dissect, discuss, distill, and discover mathematics through tactile learning. We encourage you, the curious instructor, to look through the literature to get a clearer understanding of how activities can enhance learning in your classes.

The two of us have been using a variety of tactile activities in our classes for years, and we are constantly seeking and developing new ideas. We co-organized an MAA Special Session, "Touch it, feel it, learn it: Tactile learning activities in the undergraduate mathematics classroom," for the 2012 JMM in Boston. With 35 presentations drawing an audience of 100-150 people, we decided to offer a second MAA Special Session at the 2013 JMM in San Diego, where we had an additional 26 well-attended presentations. Given the diversity of the presentations at these sessions, it is clear that there a large number of mathematics faculty members using a wide variety of tactile activities in their classes, and an even larger number of people showing interest in the subject.

In response to the high level of interest from the mathematical community, we guest-edited a special issue of *PRIMUS* [2] dedicated to hands-on activities in which the authors, many of whom have also contributed to this book, offered specific pedagogical support for the activities presented therein. We have also assembled this book, offering a diverse collection of activities targeting mathematical topics ranging in level from precalculus to knot theory. We developed this book for you, the curious instructor—both the seasoned

experimentalists and those trying hands-on activities for the first time, and we hope that you find it beneficial as you invite your students to explore mathematics tactilely in your classrooms.

We are sure that all of the people who spoke in our sessions, attended the talks, authored articles in our *PRIMUS* issue, or contributed to this book have their own tales about what initially sparked their interest in teaching this way. We include our stories below.

**Jessica's story:** My calculus class at West Point was just getting ready to transition from single variable integration to integrating functions of two variables, and I knew that my students would struggle with the concept of “the volume beneath a surface”. I spent my entire afternoon writing, scrapping, re-writing, and re-scrapping lesson plans, Mathematica notebooks, and chalkboard sketches to make the concept accessible, but nothing seemed concrete enough. The night before the lesson, I went to bed very worried. As I sleeplessly tossed and turned, I caught sight of my arm moving under the sheet, creating a beautiful surface. I sat straight up, realizing that my students could stand in formation, and by tossing a sheet over them, my class could create their own surface, using their own heights to approximate the integral. I tried this activity in class the next day, and based on the positive feedback from my students and the prodding of a colleague, I wrote an article on this activity that appeared in *PRIMUS* [6].

The year after developing the sheet activity for my calculus class, I was asked to develop a new course called Mathematics for Space Applications; there was no similar course in the country at the time. Drawing students from systems engineering, mathematics, physics, and a handful of other majors, I had difficulty designing lectures that addressed the broad backgrounds of my students. Shortly into the course, I tossed out the majority of my lectures and went to a full peer-teaching and hands-on learning approach. We represented orbits using hula hoops and globes; we made model solar systems with sidewalk chalk and string; we spun ourselves silly in office chairs as we traced out highly elliptical orbits with our feet; and most importantly, we had fun while we learned. I wrote an article on the format of this course, including the hands-on activities, for *Mathematica Militaris* [4]. Through teaching this course, I developed a new appreciation for hands-on activities as a way of leveling the playing field for a diverse group of students, and I continue to leverage this benefit in my teaching at the Virginia Military Institute.

**Julie's story:** After covering the epsilon-delta definition of continuity in real analysis, I noticed many pairs of glazed eyes staring back at me. The definition had been too abstract for the students, so I tried to explain the definition again. I drew a diagram, but it was too static. I used technology to zoom in on a graph, but students only saw the lines instead of connecting them to the original function being studied. We were all frustrated. A few days later while walking down the craft aisle in a store, I stumbled on a feather boa that had haphazardly fallen onto the floor. The way it was situated, the boa resembled a large graph of a function, and I could imagine my students walking on it while physically exploring that analysis definition. Later in class, my students were surprised when I placed a feather boa on the classroom floor. Students volunteered to use yarn to represent the epsilon and delta regions around a point. They noticed how delta was affected by epsilon. The definition came alive. Throughout the rest of the course, we used the boa or a collection of boas whenever we covered new definitions about functions. Details for that activity can be found in *PRIMUS* [1].

After successfully using feather boas in analysis, I realized that the boas could also be used in any class that studies graphs of simple functions. I gathered a collection of feather boas and developed small group activities to aid students in understanding piecewise functions as well as activities for making connections between derivatives and the shape of graphs. The added beauty of using the boas in calculus and precalculus is that students typically are intrigued by the novelty and want to touch them; this increases student participation. Also, the graphs are large enough to see from the other side of the room, making it easy to tell which groups understand the material and which groups need a little more help. Since the boas worked well for small group activities, I brought them into my senior complex variables class where students used the boas to represent the images of a basic smiley face under a variety of standard complex functions. I never realized that feather boas were such a great teaching tool in mathematics. Even more, I never realized how an idea that worked in one class could be modified to create useful activities in a wide range of classes.

We have both realized the value of hands-on teaching and learning and have adopted it in a wide array of applications. Whether presenting a challenging concept or leveling the playing field for a diverse student population, we try to develop a meaningful hands-on activity that forces our students to engage with the material, ultimately developing their own concrete understanding. Of course, the joy of seeing a room full of engaged and excited students comes at a price, as these activities require planning and take time away from lecturing. Yet, activities do not have to be done at the expense of content. Lecture time is simply replaced with the activity, and the class then has a shared experience that can be referenced for many lessons down the road. Therefore, we believe that the benefits of using activities in class are worth the investment.

Each of the activities in this book has been used and vetted by its author(s), and the write-up includes suggestions and pitfalls to help reduce that initial investment of time. A more detailed explanation of the features of this book can be found in “How to Use This Book”. We invite you to try some of the activities in this book, and we hope that you and your students both benefit from the deeper learning and the simple joy that hands-on activities can bring.

Sincerely,  
Julie & Jessica

## References

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