1

An Invitation to Mathematics for Social Justice

Gizem Karaali and
Lily S. Khadjavi

Since the 2001 publication of Bob Moses' *Radical Equations: Math Literacy and Civil Rights* [**8**], North American educators have increasingly recognized the significant connection between mathematics education and social justice. Another 2001 publication, *Mathematics and Democracy: The Case for Quantitative Literacy* [**11**], provided the impetus for mathematicians to tackle the visible and invisible roles of mathematics education in civic life and democratic societies. Today scholars from many corners of the academy are working to ensure that students from all backgrounds receive the best mathematics education they can. One way to achieve this goal is to offer our students a range of opportunities for authentic engagement with mathematics addressing issues that are relevant and timely.

As college mathematics instructors, we have the opportunity and the responsibility to help our students develop strong analytical skills and empower them to tackle complex issues. In the K-12 mathematics education literature, the theme of teaching mathematics for social justice has been present for well over a decade now. Starting with Eric Gutstein and Bob Peterson's *Rethinking Mathematics: Teaching Social Justice by the Numbers* [**5**], those school teachers keen on thinking about such issues have a number of pedagogical resources available to them. Among these is an excellent website which provides many sample lesson plans [**10**], Gutstein's thought-provoking *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice* [**4**], and a

most readable collection of essays titled *Teaching Mathematics for Social Justice: Conversations with Educators* [**13**]. All these references would be of interest to a college mathematics instructor, but as of yet, there are few dedicated resources for the college classroom. This book, together with the second volume, *Mathematics for Social Justice: Focusing on Quantitative Reasoning and Statistics*, aims to fill that gap. Indeed we hope to simply initiate this project and expect that many other resources will follow, much as a few of our own contributors were inspired by their work with Science Education for New Civic Engagements and Responsibilities (SENCER) and the Engaging Mathematics initiative.[1]

These two volumes aim to support college instructors who wish to incorporate ideas and instances of various social justice issues into their classroom by providing them with concrete examples of mathematics connected to a range of contexts. An eclectic collection of modules are accompanied by a handful of thoughtful essays on goals, methods, and possible implementation challenges associated with the idea of incorporating social justice themes into the college mathematics classroom. The primary focus of most contributions in this book is on topics where students can ask their own questions, find and analyze real data, use mathematical tools by themselves, and thus draw their own conclusions.

We have communicated with several mathematicians who have thought seriously about incorporating social justice themes into their own teaching and who had developed and used some materials for this purpose. To reach beyond those to whom we directly extended an invitation to submit, we sent out an open call for contributions and received several contributions from many others who have been thinking about similar issues. The result is an extensive, wide-ranging collection of interesting and useable materials suitable for a broad variety of college mathematics courses, touching on such compelling subjects as human rights and racial injustice.

This first volume consists of two sections:

(1) Essays: Perspectives on incorporating social justice themes into the mathematics classroom, written by college mathematics faculty experienced in this topic;

(2) Modules: Detailed and well-developed course materials ready to be adapted to a variety of courses, from precalculus to differential equations, graph theory, and beyond, written by college mathematics faculty who have often included notes about their experience in their own institutional contexts.

The second volume contains modules that fit most naturally into an introductory statistics curriculum or are specifically designed for general education / quantitative reasoning courses.

The modules typically incorporate a series of in-class activities, research assignments, problem sets, or other methods of engaging the students with the relevant mathematics involved. Most are self-contained, with background necessary to understand the context of the issue and suggestions for relevant data sources provided, so that materials can be readily included into any class where the mathematics necessary is being covered.

[1] See Engaging Mathematics at `http://engagingmathematics.ipower.com` for a stimulating collection of materials, developed with funding from the NSF, to engage students in civic issues. Most are on sustainability, for use in courses such as statistics or algebra, but others include projects related to racial justice, voting theory, and more, and many involve data collected from local communities.

More specifically, found in each module are:

The mathematical content of the module: What courses would this module work for? What are the prerequisite mathematical ideas / constructs / procedures students should know already? What will be the mathematical value of the module; in other words, what math is being taught / scaffolded / applied / strengthened?

Context / background for the module: underlying social / political / economic context, background information that students or the instructor should know.

Instructor preparation: direct instructions and additional suggestions for the instructor to be able to use the module in the classroom, possible adaptations.

The module itself: the mathematical and social situation as it is to be introduced to the class.

Additional thoughts: further investigation suggestions, how things went for the author(s) when they implemented the module.

Most modules also have appendices containing tables and datasets as well as sample handouts that instructors can use in their own classes.

Between the two of us, we have several years of professional work on the themes of social justice and mathematics, including various panels, presentations, and workshops at the Joint Mathematics Meetings, law and sociology conferences, and elsewhere, along with a handful of related papers and projects. While writing papers, reviewing journal submissions or editing journals, and making professional connections within and outside of mathematics, we had a growing desire to highlight and draw attention to the compelling and innovative work that so many are doing to bring these themes into their own classrooms. After a couple years of dreaming and planning, we are now delighted to bring this volume to fruition.

What exactly do we mean by social justice?

Before we move any further, we should probably define what we mean by "social justice" or, more specifically, by "teaching mathematics for social justice". Justice Potter Stewart's test ("I know it when I see it") will probably not suffice for an audience of mathematicians.

Following the spirit of the International Council of Scientific Unions, we use, here and elsewhere, the term "teaching mathematics for social justice" to encompass all mathematics instruction which aims for improved human well-being. Human well-being in turn is defined as a "context- and situation-dependent state, comprising basic material for a good life, freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience" [**7**].

As such "social justice" forms a cornerstone of what makes a society good; individuals and communities in such a society are all guaranteed to have certain fundamental rights. Some may offer John Rawls' theory of justice as a possible way to dig deeper into the concept. For others Rawls will not suffice; those will offer other thinkers as dependable guides. For our purposes, however, the essence is captured by the United Nations: "Social justice may be broadly understood as the fair and compassionate distribution of the fruits of economic growth" [**9**]. This, coupled with Thomas Jefferson's

incomplete list of the unalienable rights, "Life, Liberty, and the pursuit of Happiness", is a basis for our view.[2]

Teaching mathematics for social justice is an expansive project. It involves, among other things, what we do in the classroom with our students to create classroom environments where social justice is practiced. It involves exploring the foundational assumptions of our profession, as well as the historical narratives supporting or disrupting today's academic hierarchies. It also involves understanding and dismantling various structural and systemic barriers confronting large groups of students through mathematics. Recent resources such as *Inventing the Mathematician: Gender, Race, and Our Cultural Understanding of Mathematics* [**6**], *Rehumanizing Mathematics for Black, Indigenous, and Latinx Students* [**3**], and *Reflecting the World: A Guide to Incorporating Equity in Mathematics Teacher Education* [**2**], as well as the many thoughtful posts in the American Mathematical Society's *inclusion/exclusion* blog (available at `https://blogs.ams.org/inclusionexclusion/`), may guide mathematics instructors in such pursuits.

The purpose of this book is much more modest. In this volume and the next, we are almost exclusively concerned with the content of college mathematics courses. We know that this is not the only way to teach mathematics for social justice. However, we believe that incorporating social issues into one's curriculum is a low-barrier entry point into the practice.

What is in the book?

When we began work on this project, we envisioned a single book, though packed full of ideas and resources. As we reached out to the mathematics community, we found that there was much interest in and expertise with regard to this approach. The wealth of materials we gathered together could not fit into a single volume. Therefore we decided to split this collection into two volumes. Below we describe the contents of this book. The contents of the second are described in detail in the introduction to that volume.

The essays. This book starts with five reflective essays written by experienced and thoughtful instructors. These essays will encourage and support instructors in the classroom, as well as in discussions with colleagues and administrators. As such they are an integral part of this volume.

Kira Hamman in "Mathematics in Service to Democracy" shares the story of how she came to teach mathematics for social justice. Her personal story is a good place to start for instructors who may not yet be sure why they should explore social justice themes in their classrooms or how they could convince colleagues, students, and administrators that this is a worthy endeavor. As a veteran of the ongoing conversations around quantitative literacy, Hamman brings to this volume a strong and well-supported conviction that mathematicians need to take more responsibility in the education of the next generation, and that teaching mathematics for social justice is a concrete way to do so.

Suppose you are a mathematician convinced that it is a good idea to try to bring social justice issues into your classroom; after all, you are already reading the introduction

[2] If we were to follow this line of thought, we should like to incorporate into a comprehensive definition the individual-directed as well as the communal-directed rights of all people. Political philosopher Danielle Allen argues that these are also accounted for in the American Declaration of Independence; see [**1**].

to a book just about that! Still you might not feel equipped to jump in. Most mathematics instructors have not had much experience leading in-class discussions, especially those where there may not be clear and objective answers. You might wonder about the pitfalls and the myriad ways things can go wrong once you take the plunge. In her essay titled "Preparing for Student Resistance: Rules of Engagement for Sensitive Topics", Lisa Marano explores just these kinds of issues. In particular, we read about the many ways students may resist this approach and a handful of other unfamiliar teaching problems that mathematics instructors may now need to consider. Marano pulls examples and perspective from a course which she developed and has taught for many years. From her experience the reader will gain confidence that the challenges of teaching mathematics for social justice are surmountable.[3]

For many years there has been a lively effort within the mathematics community to incorporate into standard curricula mathematical problems that involve environmental issues. In the essay "Social Justice and Sustainability: Two Perspectives on the Same System", Jason Hamilton and Thomas J. Pfaff argue that that movement should not be independent from the math-and-social-justice movement. More broadly, they argue that the problems facing the current and the future generations on this planet are complex and multifaceted, and they require sustained and nuanced thinking that can clearly make the connections between environmental and social justice issues. We agree. To this end these two volumes contain a handful of modules that revolve around the issues of environmental justice. See in particular the contributions of Archey; Cohen and Pivarski; Henderson and Kose; and Zobitz, Bibelnieks, and Lester in this volume, and Galanthay and Pfaff, and Piatek-Jimenez in the companion text *Mathematics for Social Justice: Focusing on Quantitative Reasoning and Statistics*.

This volume also contains an essay from Victor Piercey who describes a theoretical construct he labels "quantitative ethics". The essay, also titled "Quantitative Ethics", is an exploration into this construct. According to Piercey, quantitative ethics is inquiry into the ethical implications of generating, using, and disseminating quantitative data. We are not only interested in understanding others' data, Piercey tells his students. We also want to make sure that when we ourselves create data or analyze data and present our numbers to others, we are cognizant of the ethical implications of what we are doing. Piercey offers some provocative examples, and there is much to be pondered upon here.

The last essay, titled "Math for Social Justice: A Last Math Class for Responsible Citizens", and written by Dave Kung, is about a semester-long course Kung has developed and taught for many years that explicitly explores and builds connections between mathematics and social justice. Most of the modules we collected together in this volume are designed to fit into already extant and relatively standard course syllabi. Kung has chosen a different path and developed a whole course. In his essay, he describes his reasons for creating this course and shares some of his experiences with teaching it. Instructors who are intrigued by the challenge will appreciate the contributions of

[3]Module contributors to this volume also put in significant effort into providing support for instructors who might be interested in using their classroom materials on various issues that may come up. To this end, for instance, several modules have sections on specifics of grading this kind of work; others provide some hints on how best to assign and help support students through projects that have components that might be unfamiliar to a typical mathematician, such as writing prompts and poster presentations. Furthermore each module contains a section on the social and political background of the issue under exploration for the instructors who might otherwise feel unprepared about diving into the topic as non-experts.

Beier in this volume and of Franco and Piercey in the companion book *Mathematics for Social Justice: Focusing on Quantitative Reasoning and Statistics*; each of these modules describes a semester-long course built around social justice.

The modules in this book. The bulk of this volume consists of fourteen modules, exploring a wide variety of social justice themes and offering the opportunity to include them in courses ranging from college algebra to discrete mathematics. Below we give a brief description of each one, to give the reader a feel for the topics and the mathematics involved; tables provided in the next section will allow the interested reader to track down particular subjects of interest.

Dawn Archey in her contribution titled "Sea Level Change and Function Composition" describes a module intended for a college algebra or precalculus course. Basic mathematical ideas such as function composition and similar triangles come in handy as students work to model the effects of global climate change on small island nations. Numerous supporting references regarding climate change are provided, including those for predictions of change in sea level. The main part of the module can be completed in class; there is also a follow-up writing activity which can be assigned as homework. Archey provides a student handout as well as extensive data to support the instructor.

In "Exploring the Problem of Human Trafficking", Julie Beier has designed a module centered around graph theory and the issue of human trafficking. The module makes use of simple constructions from graph theory to allow students to engage with sophisticated problems. Spanning over eleven class periods, it can be modified to fit shorter time limits and may be used with students with any mathematical background, as the prerequisites, both for the social context and the mathematics involved, are built into the project described. Students gain a deeper understanding of the problem of human trafficking, are introduced to network theory, and develop an appreciation for how mathematics can help us tackle challenges in today's world.

Geoffrey Buhl and Sean Kelly present students with the challenge of creating voter districts which are deemed fair in their contribution titled "Evaluating Fairness in Electoral Districting". The issue of gerrymandering provides the political context for this module, originally designed for a general education course but also adopted in an environmental science workshop. With hands-on materials, students create districts and then mathematically evaluate the fairness of their construction, given geometric compactness measures and an index for measuring voter power. Buhl and Kelly provide background for these types of measures, as well as structured handouts and discussion questions. Given the contributions of mathematicians such as Moon Duchin to related conversations (see [**9**] for recent coverage of her work), this module may be a great segue to other mathematical topics as well as more engaged discussions of mathematics and its role in the public sphere.

In "Modeling the 2010 Gulf of Mexico Oil Spill", Steve Cohen and Melanie Pivarski describe a module where students explore the environmental disaster caused by the catastrophic BP oil spill from an explosion on the Macondo well rig. The mathematical tools required include area computations using integration, and as such, the module is perfectly suited for a semester-long project in an integral calculus course, though Cohen and Pivarski offer suggestions for adapting it for a wide range of courses that cover

differential equations and mathematical modeling. This is one of a handful of modules included in this volume exploring environmental issues; as argued eloquently by Jason Hamilton and Thomas J. Pfaff in their essay titled "Social Justice and Sustainability: Two Perspectives on the Same System", such issues are directly related to social justice. However Cohen and Pivarski also point out that working with real world data to investigate global corporations and their actions can empower students who might otherwise feel at sea in confronting faceless corporations which may seem invincible.

John Cullinan and Samuel Hsiao present a module suitable for discrete mathematics courses in their contribution titled "Voting with Partially-Ordered Preferences". With few mathematical prerequisites, the project may also be used for appropriate general education courses. Indeed the authors have used it in a course on mathematics and politics. The main mathematical ideas in the project are partial orders and voting theory. The central political question posed in the module is rather sophisticated and is bound to engage students. The design of the optimal political system is a complex question, and how best to represent voters' preferences is an unsolved (and perhaps unsolvable – see Arrow's Theorem mentioned in this module and elsewhere in the volume) problem. This module is one of a handful in this volume on elections and voting.

John Curran and Andrew Ross describe a role-playing game in their contribution titled "Implementing Social Security: A Historical Role-Playing Game". In this game, designed for a quantitative reasoning course, students reimagine and enact a fictional version of the historic debates in the United States Congress which eventually led to the passage of the 1935 Social Security Act. This project is an extensive one, and was originally developed for the Reacting to the Past (RTTP) consortium. In their contribution to this volume, Curran and Ross provide an overview of the project and the extensive resources freely available elsewhere for interested instructors. The module allows students to explore a complex social issue through the perspective of various constituents of a democracy and use mathematical tools to make convincing arguments to support their positions.

"Matching Kids to Schools: The School Choice Problem", by Julie Glass and Gizem Karaali, introduces a two-sided matching problem that confronts the challenge of finding fair ways to distribute students among the schools in a large school district, given constraints on student preferences and priorities such as proximity, special programs, and sibling placement. The module has no prerequisites and can be used in a course on discrete mathematics, or, with ample guidance, in a liberal arts mathematics setting. Throughout the project students are encouraged to consider what makes a matching algorithm fair and seek ways to encode these expectations in mathematical terms. Several exercises with solutions are provided.

In "Modeling the 2008 Subprime Mortgage Crisis in the United States", Bárbara González-Arévalo and Wanwan Huang describe a group project designed for a financial mathematics course. This module revolves around the 2008 mortgage crisis which caused an upheaval of the housing market in the U.S., affecting the lives and livelihood of many people. Students use real data to analyze the causes of the crisis and its effects on specific populations. The semester-long project ends with a poster session and involves students engaging with the local community, providing a neat way to incorporate civic engagement into a mathematics course.

Bárbara González-Arévalo and Wilfredo Urbina-Romero have designed a semester-long project for a second-semester calculus course in their contribution titled "Using

Calculus to Model Income Inequality". At the center is the Gini index, perhaps the most well-established measure of income inequality; in the module, students learn about the Gini index and then work to pose a good research question involving it. Studying the mathematics of the Lorentz curve, curve fitting, and numerical integration techniques, and collecting relevant data for their project, they develop the necessary expertise to investigate their research question. They wrap up the semester with a written report or a poster presentation. Directions that can be used to prepare suitable handouts are provided in the appendices. This is among a handful of modules revolving around the notion of income inequality.

Another contribution on elections and voting where Arrow's Theorem shows up is Kira Hamman's "What Does *Fair* Mean?" which describes a project that is specifically designed for a quantitative literacy course. In this module, students explore various voting methods and in the course of a class hour build up to Arrow's Impossibility Theorem. Hamman embeds the mathematical component of the project within a whole-class discussion of just what is fair. She provides suggestions for discussion prompts as well as an optional spreadsheet assignment for instructors who wish to explore the mathematics further.

Another module that draws attention to the social justice implications of environmental issues is "Social and Environmental Justice Impacts of Industrial Agriculture", by Amy Henderson and Emek Köse. In this extensive module, students work with mathematical models using ordinary differential equations to study the environmental impact of the waste created by industrial livestock farms across the United States. The material is suitable for courses in differential equations and mathematical modeling, and the authors provide handouts and instructor solutions for self-contained work, as well as resources which could support a multi-week project.

Reem Jaafar in "Student Loans: Fulfilling the American Dream or Surviving a Financial Nightmare?" presents a three-part module revolving around student debt, a topic that will be relevant to many college students. The main mathematical content focuses on polynomial models and their limitations. Therefore this project could be suitable for general education or quantitative literacy courses, as well as early parts of courses in mathematical modeling. The author provides a reading list for students and several handouts for the instructor on various components of the module.

Angela Vierling-Claassen's contribution titled "Modeling Social Change: The Rise in Acceptance of Same-Sex Relationships" introduces social network theory to explore social change. Students develop and then analyze a model that represents a social network where people's connections are strengthened or weakened progressively along with changes in their attitudes toward same-sex relationships. An optional computer-aided extension is also described. The mathematics involved is basic graph theory, and there are few prerequisites, so the module can provide a mathematical experience for students with widely differing backgrounds enrolled in discrete mathematics courses as well as general education or mathematics for liberal arts courses.

John Zobitz, Tracy Bibelnieks, and Mark Lester's "Sustainability Analysis of a Rural Nicaraguan Coffee Cooperative" grew out of an experience with a cooperative during a study abroad program and an Engaging Mathematics initiative, funded by NSF. The module exposes students to mathematical modeling in an introductory calculus course, motivated by researching the actual resources and concerns of a Central American coffee cooperative to promote sustainable practices in ecotourism. Students learn

to apply ideas about rates, optimization, and modeling, with contextual resources including short introductory videos from the cooperative, guided homework analyzing basic revenue models, and the opportunity to analyze data from Google trends.

How to use this book

The essays in the book, which make up Part B, are organized in what seems to us a logical way. We expect that reading them in the order presented here will feel natural to most readers.

The modules, on the other hand, were harder to organize. We first thought about grouping them based on the courses they fit best. However as most modules would fit well into a range of courses rather than a specific single course, this ordering was somewhat imprecise. We considered the other thematic alternative, ordering the modules in terms of the socio-political issues they tackled. This too seemed somewhat ad hoc. In the end we grouped the modules into two separate volumes, each suitable for a range of courses, but with the more statistical or quantitative-reasoning approaches in *Mathematics for Social Justice: Focusing on Quantitative Reasoning and Statistics*.[4] We urge readers to skim through both books and be open-minded about what sort of mathematics will fit within what kinds of courses.

To help readers better navigate the contents of this volume, we have created two tables, presented in the following pages. The first table categorizes the modules by mathematical content and types of courses they would be appropriate for. The second aims to classify the modules in terms of the social justice issues that are explored within.

Our assignment of the modules into courses listed in Table 1.1 is merely suggestive. We hope that readers will explore all modules that might be mildly relevant to the class they are preparing to teach. Similarly, we have clustered the modules by social justice issue in Table 1.2, grouping the myriad issues into seven broad categories: *access*, *citizenship*, *environmental justice*, *equity/inequity*, *finance*, *human rights*, and *labor*. We hope that these tables will help readers get their bearings and feel comfortable exploring the contents of the volume.

While the tables provide an organizational guide to this eclectic collection of resources, we nonetheless believe that the best way to work with this book is by flipping through the pages, reading the abstracts, and skimming through the handouts in the appendices to see what might work for your classes. Through their efforts, the contributors to this volume have drawn compelling issues into the mathematics classroom and challenge us to do the same.

What next?

All our contributors are keen on sharing their work and will be happy to hear from you. If you are interested in any of the modules you find here and want to know more, or if you have any ideas about adopting one for your own context and would like to have access to source codes, TeX files, and so on, please feel free to contact the respective authors.

[4]See the two tables in the Postscript for more information on the content of *Mathematics for Social Justice: Focusing on Quantitative Reasoning and Statistics*.

We believe that this book is only a first step towards a broader goal: to establish an active and ongoing engagement of the college mathematics community with social justice issues. In order to instill the ideas of social justice into various introductory and general-education mathematics courses, we need to encourage the development of more teaching materials. It will take more effort for these kinds of concerns to influence the mathematics classrooms which center around more advanced material; we urge instructors to think creatively about how this can be done in courses such as abstract algebra, real analysis, and differential geometry. In the near future we hope to create an accompanying website which will contain up-to-date information about these modules, and links to resources and datasets. We hope to include in that site other modules as well, and we encourage readers to think about developing their own.

Last but not least, we need to bring to the broader mathematical community critical conversations about mathematics and its diverse and divergent roles in the social and public domains. Teaching mathematics for social justice is indeed much broader and more multifaceted than simply incorporating some social-justice themed modules into one's classroom. As we said: this is only the beginning.

A note on the cover art: The raised fist has a long history as a symbol of solidarity and of resistance against various forms of oppression. In this book cover, designed by Courtney Rose, the figure represents the possibilities of mathematics to empower us with a view towards creating a better world.