Teach Them to Fish

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Give people a fish, and you feed them for a day.
Teach people to fish, and you feed them for a lifetime.
—Proverb; after Anna Isabella Thackeray Ritchie

Every year, nearly 3 million people enter college in the United States. More than one-third of them plan to study science, technology, engineering, or mathematics (STEM), and 35 percent of the students intending to major in STEM belong to racial and ethnic groups underrepresented in the sciences, including African Americans, LatinX, and Native Americans (NCSES, 2017). What an amazing opportunity! Hundreds of thousands of people, reflecting the rich diversity of America, come to college wanting to participate in STEM fields. Regardless of who among them eventually completes a STEM degree, all are poised to participate in our society as voters, as parents, as teachers, as writers, as scientists, as members of Congress.

Unfortunately, most of the students who enter college wanting to earn a STEM degree do not. For students intending to major in a STEM discipline, the five-year STEM baccalaureate completion rate is approximately 40 percent for all students, and only 20 percent for students from underrepresented racial and ethnic groups (National Academy of Sciences, National Academy of Engineering, and Institute of Medicine, 2011). Regardless of race and ethnicity, most of the leaving occurs early in the undergraduate experience.

How do we squander this amazing opportunity? Instead of celebrating creativity and innovative thinking—attributes that we know are necessary ingredients of STEM—too often our courses emphasize the memorization of "facts." Instead of creating activities that encourage students to explore the unknown, too often we insist the students perform exercises for which the outcomes are already known. Instead of celebrating the enormous strengths residing in the diverse group of students who want to learn STEM, too often we focus on "weeding out" those whom we deem unfit. While it would be naïve and disrespectful to think that everyone who expresses an initial interest should complete a STEM degree, neither should we actively alienate our students from STEM. The problem is not that students leave STEM. The real tragedy is that we don't take better advantage of the opportunity to introduce our students to the processes of science and mathematics *before* they leave STEM.

College should be the time when students explore and experiment, and engaging undergraduate students in authentic research is a good way for them to gain an understanding of the process of discovery, experiencing how research outcomes lead to evidence that informs decisions. Because undergraduate students are relatively new to the discipline it is important to carefully construct a framework in which they can build expertise incrementally, with modest early steps leading to later activities that are more ambitious. Although style is likely context-specific and varies among disciplines, all disciplines—STEM and non-STEM alike—can be home to effective undergraduate research.

It is one thing to accept in abstract terms the value of undergraduate research experiences, but it is a very different matter to actually make it happen and happen well; this is a daunting challenge for the prospective mentor. After all, most of us were trained to think about our research, not about how to translate our research into learning experiences for undergraduates. But we must try to realize the promise of undergraduate research.

A Mathematician's Practical Guide to Mentoring Undergraduate Research is a vital resource for faculty in mathematics who are the primary instructors and research mentors for the M in STEM. As the majority of STEM degree programs include mathematics coursework, however, this new book is relevant to all STEM disciplinary areas and provides several important tips to prospective mentors, including consideration of these four questions:

(1) How can the mentor guide the choice of the research problem so that the discoveries made by the students advance the knowledge in a field of inquiry while also achieving positive learning outcomes? In mathematics, this has been a particularly daunting challenge, with many believing that formulation of good undergraduate research problems in math is impossible.

(2) How can the mentor create an inclusive environment that promotes student self-efficacy, especially for those who come from underrepresented groups? Again, the perspective of mathematics provides important insights because diversity and inclusion have been difficult issues in the discipline.

(3) How can the mentor design the experience so that the students develop the important personal skills of self-reflection and effective listening, and the social skills of teamwork, collaboration, and communication?

(4) How might the mentor apply guidelines such as the Center for Undergraduate Research's Strategic Pillars (CUR, 2012) to offer authentic undergraduate research experiences in the future?

I don't know what STEM will be in 20 years, but I am certain that it will be different. There will be hot new topics and other topics that will have cooled off. There will be new disciplines emerging at the intersections of traditional disciplines. The way we teach will also be different, with new ways to engage students in active learning, in early and authentic research experiences, and in the effective use of online tools.

As society accelerates into the future, we have a duty to ensure that the learning in STEM is inclusive, so that no one is left behind. Our aspirations should be high and not apologetic. Our approaches should be based on responsibility and not on deficit-thinking. Our actions should be intentional and measurable, and not an afterthought. Our goal should be to teach our students to fish.

Give students a fact, and they might remember it for a day.
Teach students the process of discovery and
they will be prepared for a lifetime of learning.

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Foreword

by Francis Su

You may be reading this book with some skepticism. Perhaps you've never done research with undergraduates before, and you wonder: is undergraduate research really possible, in theory? Or you have tried it, but you have encountered obstacles that cause you to question whether undergraduate research really is doable, in practice. Maybe you are unsure about whether you are well-suited to supervise such research. Almost certainly you're a busy person, and among the countless good things you could be doing, you ask: should undergraduate research be a priority? The good news is: you're reading the right book to answer those questions, and they will be answered in the affirmative.

More importantly, this book may cause you to ask a different question: what kind of mathematical experiences do I want my students to have?

My own relationship with mathematics came alive by doing research as an undergraduate. I didn't have the benefit of doing a summer program like the ones that exist today, but I worked with a professor during one school year who gave me a real research paper to read and a question to think about.

I remember thinking how unintelligible the paper was and how hard it was to slog through it, but it was confidence-building that the professor had entrusted me with a problem and exciting when I made some incremental progress. But beyond the results, doing research was just as valuable for all the hidden mathematical skills it was nurturing in me.

For instance, I began to engage my classes differently—more actively—because I would pay special attention to things in my courses that might help with the research problem I was thinking about. Suddenly, everything began to look connected: just like when you learn a new word for the first time and then you begin to read it everywhere.

And I began to read math books differently—more actively—because my experience in reading a research paper helped me see that when someone solves a problem the first time, they don't necessarily see the most elegant solution or the path to the solution that I would take. Thus, I learned through doing research as an undergraduate that I have something valuable to contribute when I read any work of math—my own way of seeing things. I began looking at every math text I read and asking: how would I have explained this material in my own way? And that skill served me well once I began to teach.

Do you want your students to build their own sense of agency so they realize that they have something to contribute to their own education? Do you want them to be more actively engaged in all their classes? Do you want them to grow their own mathematical identity? Do you want them to build enough confidence so that their identity is unassailable when someone undercuts them later in life? Point them to a real research problem. Do you have to be an expert in that problem? No. You have the soft skills to coach them and help them take ownership. In fact, one of the joys of research, I tell my students, is that they get to become the local expert on a problem.

The impact undergraduate research can have on a college student cannot be overstated. Which means undergraduate research can be most valuable for students who are not yet actively engaged in mathematics. And course grades don't often measure many of the skills that are cultivated in research, like persistence in thinking about a problem over a long period of time.

So don't just point high-performing students towards real research. And be cautious about making assumptions about who will want to do research. Because there is mathematical potential to be unleashed in every student.

This book has so much thoughtful, practical advice about doing research with undergraduates, gathered by authors who have been nationally recognized as great teachers as well as leaders in the movement towards undergraduate research. Both novices and seasoned faculty will find something valuable here. If you follow the advice in this book, you have the potential to completely transform the mathematical experience of many of your undergraduate students.

In so doing, your own mathematical experiences will be transformed, too. The dual rhythms of the academic life—teaching and research—will feel united towards a common purpose. Some of my best experiences in teaching have come from doing research with undergraduates. Many of the former students that I remain closest to—indeed many of them are math faculty now—are ones that I co-authored a paper with when they were undergraduates. After having such experiences yourself, you will find that you will want to prioritize research with undergraduates because it will lead to some of the richest teaching and mentoring experiences you can have as a faculty member.

Enjoy the adventure!

About this book

This book is a practical guide for mentoring undergraduate students in mathematics research. Although there have been many individual articles about undergraduate research in mathematics, some dedicated journal issues, edited books, and numerous conferences and workshops, there are few resources that collect information on all the facets of directing undergraduate research in one convenient place. Whether you're new to undergraduate research and looking for advice on how to start or you're a seasoned mentor of undergraduate research projects searching for new ideas and inspiration, this book provides a resource to help you.

The primary audience of this book is mathematicians and faculty who want to learn how to plan, start, implement, lead, conclude, and assess various types of undergraduate research. There are sections on the fundamental aspects of mentoring students in research, from choosing research problems that are well-suited for student success (Chapter 3) to helping students present their research results (Chapter 5). The book also discusses less standard, but important topics, such as how to manage a variety of student personalities that may affect the dynamics of a research group (Sections 4.3, 4.5) and working with students from underrepresented groups (Section 4.4—this section was written by faculty from underrepresented groups who are experienced in doing undergraduate research). Beyond the nuts and bolts of working with undergraduates on research, this book provides suggestions for those who want to move further ahead with undergraduate research by writing grant proposals and getting funding for doing undergraduate research (Chapter 6). Many readers will be interested in thinking about where undergraduate research is headed and where there are needs for people to make new contributions. We provide some of our thoughts on these ideas in Chapters 8 and 9.

In addition to being a resource for math faculty, this book has sections that may be of interest to others. Undergraduates interested in doing research will find portions of this book helpful, such as the benefits to students of doing research (Section 1.2), the key things that faculty know about research that their students probably don't know (Section 2.5), what makes a good research problem (Chapter 3), how to work in a group of students doing research (Sections 4.3-4.5), how to present research results (Chapter 5), what an REU is (Section 7.1), what careers there are in industry for math students (Section 9.3), and what some other models for doing undergraduate research are (Sections 9.2, 9.4, and 9.5).

Directors of undergraduate research programs at universities may also benefit from having a copy of this book. Unfortunately, it is not uncommon for advocates for undergraduate research to hear that undergraduate research in mathematics is nearly impossible to do. Our experience shows that this is not true. We recognize, however, that to successfully collaborate with undergraduates in mathematical research requires mathematicians to view working on research with students from a different perspective than they might view working on research with colleagues. This book provides insight into this different perspective as well as practical guidance that directors of undergraduate research programs can use to help math faculty who would like to get involved.

This book is based on years of both our own experiences as well as things we've learned from countless resources and conversations with other experienced practitioners. We'll point you to many helpful resources as we go, though you'll find that many of our examples come from personal experience. As you read, you might wonder, "Who are Michael, Allison, and Lara?"

Allow us to introduce ourselves.

Michael Dorff is a Professor in the Mathematics Department at Brigham Young University. In 2007, he founded the NSF-funded CURM program, the Center for Undergraduate Research in Mathematics. From 2007-2017 he directed CURM, supporting 400 undergraduates and 110 mathematics faculty at 84 institutions nationally to do undergraduate research. During each of those summers he ran a three-day CURM summer faculty training workshop on undergraduate research. Michael is also co-founder and co-director of PIC Math (Preparation for Industrial Careers in Mathematical Sciences), an NSF-funded program sponsored by the Mathematical Association of America (MAA) to prepare students for industrial careers in the mathematical sciences through training faculty to work with students on research problems from industry. From 2014-2017, 1400 undergraduate students at over 100 institutions have worked on research problems from industry. He has given about 50 presentations to faculty on successfully mentoring undergraduate students in research and he has mentored over 50 undergraduate students in his own research, including during an NSF-funded Research Experience for Undergraduates program that he directed at Brigham Young in 2005-2012. Michael is the 2019-2021 President of the MAA. He is a former councilor and Executive Board representative for the Mathematics and Computer Sciences Division of the Council on Undergraduate Research, a 2010 winner of the MAA's Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching of Mathematics, and a Fellow of the American Mathematical Society.

Allison Henrich is a Professor of Mathematics at Seattle University. She has mentored more than 30 undergraduate research students in knot theory projects in the Williams College SMALL REU, the University of Washington REU, in Seattle University's own SUMmER REU, and in academic year research at Seattle University. From 2015-2017, Allison was the PI and co-director of Seattle University's SUMmER REU, a summer program which aimed to broaden undergraduate participation in mathematics by targeting students in their first two years of college, including students from community colleges. She is a Red '08 Project NExT fellow and has served as a councilor in the Mathematics and Computer Sciences Division of the Council on Undergraduate Research since 2014. Allison won the MAA's Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member in 2015.

Lara Pudwell is an Associate Professor of Mathematics and Statistics at Valparaiso University in Valparaiso, Indiana. She received her undergraduate degrees from Valparaiso and then earned her Ph.D. in Mathematics at Rutgers University. Her research is in enumerative combinatorics, and she is passionate about involving undergraduate students in her work. At Rutgers, Lara spent three years as the graduate assistant for the DIMACS REU. At Valparaiso, she has co-directed the NSF-funded Valparaiso Experience in Research by Undergraduate Mathematicians (VERUM) program since 2010. Between VERUM and projects with Valparaiso students during the academic year, she has mentored more than 35 students in research. Lara is a Red '08 Project NExT fellow and a former councilor of the Council on Undergraduate Research (CUR). She was recognized by the Mathematics and Computer Sciences Division of CUR in 2013 for Outstanding Mentoring of Undergraduate Research, and she won the MAA's Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member in 2014.

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Why should undergraduates do research?

1.1 Introduction

Since you've picked up this book, it's possible you have a good idea of what undergraduate research is. You may have experienced undergraduate research in a variety of settings—as an undergraduate, as a graduate student TA, or as a research mentor. On the other hand, you may only have a vague idea of what undergraduate research is, especially if you've never experienced it yourself. Since it may not be clear what the term "undergraduate research" encompasses, we propose a working definition so that we can all begin on the same page.

From our perspective, undergraduate research in mathematics typically refers to the collaboration of one or more mathematicians (e.g., math university faculty members or math graduate students) with one or more undergraduate students (mathematicians-in-training) in order to create new knowledge in a mathematical field. This research may take place in large or small groups. Perhaps the team works for a few hours each week during the academic year or they work full-time in the summer. The results may be publishable in highly ranked disciplinary journals; on the other hand, what you and your students discover might not be appropriate for publication at all. The work may require very advanced undergraduate collaborators who have an unusually strong mathematics background, or the work may be approachable by students in calculus.

Regardless of what an undergraduate research experience looks like for you, it is clear that the experience must involve some effort on your part. So why do it? You're likely a very busy person who already doesn't have enough time to teach your classes, do your research, and fulfill a dizzying array of service responsibilities as fully as you would like, not to mention spend time with your family and friends, exercise, get enough sleep, do chores around the house, and pursue hobbies. Fortunately, doing research with undergraduates doesn't require magically enhancing each day by the inclusion of a few extra hours. With a shift in perspective, undergraduate research may require no more time than you currently spend on your scholarship and mentoring of students, but it may provide you with greater job satisfaction by connecting these two enjoyable aspects of your job, and it will provide your students with a much richer learning experience.

1.2 Benefits to students

For your students, the benefits of engaging in research are tremendous. Students not only gain knowledge of mathematics and skills pertaining to research, but they rise to a new level of academic achievement. Their professional growth and advancement is immeasurable, and they experience great personal growth in the process [**170**]. The Council on Undergraduate Research (CUR) is a strong advocate for undergraduate research, and CUR has many publications that discuss these benefits [**37**]. Specifically, studies [**15**, **118**, **125**, **196**] have shown that students who are engaged in undergraduate research are more successful during and after college in terms of problem solving, critical thinking, independent thinking, creativity, intellectual curiosity, disciplinary excitement, and communication skills. Also, undergraduate research is a "high-impact practice" that is positively correlated with higher student GPAs, increased retention rates (especially during the first and second years of college), increased student graduation rates, higher student satisfaction with college, and an increase in the number of students pursuing graduate degrees [**9**, **21**, **111**, **215**]. For students from underrepresented groups, a research experience with an experienced faculty mentor is positively correlated with improvements in students' grades, retention rates, and motivation to pursue further education and succeed in graduate school [**14**, **15**, **18**, **104**, **142**]. With respect to mathematics, the Mathematical Association of America (MAA)'s Committee on the Undergraduate Program in Mathematics (CUPM) has a subcommittee that focuses on research by undergraduates. They produced a 2006 report entitled "Mathematics Research by Undergraduates: Costs and Benefits to Faculty and the Institution" [**133**]. The report states:

> Students receive tremendous benefit from this activity. Students get to be involved in a significant mathematics project under close supervision by a professor. They gain experience with independent learning, a skill that will prepare them for research in graduate school as well as prepare them to be productive members of a company. They get control over their education in ways that are impossible to duplicate in the classroom environment. Students come out of this experience significantly enriched in their understanding of modern mathematics. Presentation of the results in written and oral formats improves the communication skills of the student.

Research projects for undergraduates can help students prepare for graduate school in mathematics, but they also provide great experiences for students who want careers in business, industry, and government. Mathematicians working in industries that hire mathematics students often say that working on an unsolved problem in a group in the setting of an undergraduate research experience or an internship is one of the top things mathematics students should do to get a nonacademic job. This experience helps students make the transition from structured coursework to open-ended research. Some undergraduates are very good at studying course material and working on difficult textbook problems that require several hours to solve, and yet they still have difficulty making the transition to graduate school or to their first post-graduate job. Although the former ability provides foundational background in mathematical content, it is a different skill than the one used doing research, which requires longer periods of work on a single problem without many inherent clues on how to prove results. Typical classroom experiences provide students with even less experience in trying to prove or disprove something the truth of which is unknown. Being able to stick with a challenging problem amidst doubt, frustration, and temporary, but deep, feelings of incompetence is an invaluable skill. Working on research problems early on—particularly in a mentored environment where experienced researchers can help students navigate mathematical and emotional obstacles—can help undergraduates make a successful transition to graduate school and to careers.

The 2014 Gallup-Purdue Index Report [**81**], derived from a study of over 300,000 college graduates in the US, reinforces these points. The study aims to determine which factors lead to higher long-term career and life satisfaction. They found six key indicators:

- I had a mentor who encouraged me to pursue my goals and dreams.
- I had at least one professor who made me excited about learning.
- My professors cared about me as a person.
- I had an internship or job that allowed me to apply what I was learning in the classroom.
- I was extremely active in extracurricular activities and organizations while attending college.
- I worked on a project that took a semester or more to complete.

Each individual indicator has a strong positive correlation with alumni feeling more engaged in their post-college workplace, and the impact is strengthened for students who agree with two or more of these indicators. However, only 3% of the students surveyed strongly agreed with all six indicators. Undergraduate research is a powerful way to address most of these areas, leading to further satisfaction when students graduate and join the work force.

In general, the ability to work over an extended period of time on a messy, unsolved problem is a skill that will benefit nearly any professional. When students gain experience in solving open problems, they gain confidence in their own ability to handle challenging tasks which, in turn, can lead to greater career satisfaction. Doing research requires a great amount of perseverance, another quality we can hone in our students that will pay large dividends for them in their lives regardless of the path they decide to travel.

1.3 Benefits to faculty

What about benefits to you, the research mentor? Long term benefits depend, in part, on your current academic setting. If you are at an institution with an existing culture that highly values undergraduate research, you are in luck! Mentoring students will help you strengthen your case for tenure and promotion or contract renewal and allow you to contribute to this wonderful environment. On the other hand, if you are at a school that does not yet have such a culture, but that places a high value on the education of its students, you have an opportunity to become a leader and a catalyst for positive change. In this role, you can demonstrate the benefits of undergraduates conducting research with your own mentoring experiences to exemplify this kind of engagement.

This is the big picture, but what benefits does mentoring undergraduates in research provide you on a monthly, weekly, or even daily basis? Here are just a few possibilities:

- *Sharing a passion for mathematics.* If you are a productive researcher who absolutely loves the work you are doing, then one of the best ways to share your joy of discovering new mathematics is to involve your students in research. In a classroom setting, students may detect your enthusiasm for mathematics, but they will typically not be exposed to the pleasures and challenges of producing original results. Working in a small group on a specific project gives students a much more authentic sense of the creative essence of doing mathematical research, and, even better, they learn about this process from sharing the experience with an enthusiastic practitioner.

- *Gaining a new perspective.* Meeting with students regularly will offer yet another outlet to think collaboratively about open problems. In fact, since students have far less mathematical preparation than a well-seasoned researcher, they are unencumbered by the "usual" way of approaching problems and may offer fresh insights into your work. Similarly, along the way, you may need to teach students material that you have known for a long time. Finding an appropriate way to convey these ideas to your students and addressing their questions may also give you a new and helpful perspective on old, but relevant, material.

- *Going beyond the curriculum.* To help students get started on a new research project, you may need to teach them about topics that do not appear in a typical classroom setting at your school. For instance, there are very few upper-level mathematics course offerings at many smaller, primarily undergraduate institutions. Perhaps you never get to actually teach a course in algebraic topology or functional analysis. Even if your department does offer a course in your general research area, chances are that the course doesn't spend much time on your particular research topic. Whatever your students' research project, you will be afforded a great opportunity to help them learn the fundamentals of the field. Chances are, teaching this material will not only help keep you sharp on details that can fade with time, but will also be an appealing change of pace from the usual lower-division teaching assignments.

- *Getting reenergized.* Perhaps you are someone who has fallen out of love with your own research, or maybe you simply haven't been able to make time for scholarship in quite a while because of the other demands on your time. Once you've fallen off of the research wagon, it can be hard to get back on. Doing research with undergraduates—either in the field in which you wrote your Ph.D. dissertation or in a completely different field—could be a great way to enjoy regularly engaging with scholarship again. Giving students an authentic research experience will inspire you to learn about the current research in a particular field. You will certainly spend some time thinking about questions related to your students' work, even if this pondering is primarily confined to your research meetings. You will likely meet with your students at least once a week to discuss research, which will keep open mathematical problems near the forefront of your mind. You and your students will hold each other accountable for moving your project forward.

- *Building mentoring relationships.* Regardless of your specific motivations for collaborating with undergraduates, the benefits of mentoring student researchers are nearly universal. Most experienced faculty can attest that the relationships they have formed with their research students are stronger and more satisfying than those they have formed with students in their classes. Through collaboration, you get to know your students as people and see promise in them that might be missed if they are simply one among many students in a class. You have the opportunity to challenge and encourage your research students as individuals as you work with them one-on-one or in small groups. Typically, you will interact with them for longer than a single term during the school year or more intensively in the summer. In fact, some of these mentoring relationships can last for a lifetime!
- *Cultivating personal satisfaction.* The sum of all these benefits is even greater than the individual parts. Working with undergraduates on a research project is an excellent way to ensure your week includes a block of engaging mathematical discussion. When you are reenergized and have a regular opportunity to share your passion for mathematics with dedicated students through mentoring relationships in a small group, it is likely that you will have deeper satisfaction in your work, both with your research students and away.

When the topic of benefits for faculty is discussed, the companion topic of incentives or rewards for faculty often arises. Two ways some institutions have rewarded faculty members for their mentoring efforts are: (1) extra funds for the faculty member and (2) a course reduction or teaching credit. Sometimes, a university can offer internal funds to reward faculty members, but with tighter budgets, this is less common. However, there are some external funding resources such as PIC Math, CURM, and NREUP (see Chapter 6 for more details). With respect to course credit, some universities or departments have implemented a plan for the faculty member to receive fractional credit for mentoring one student (such as 0.2 credit hours per student per term). A professor who has enough students working on research may earn sufficient credit to get a course reduction. This works particularly well for disciplines in which faculty can mentor a large number of students in a group. In mathematics, it is more common for a faculty member to work with a smaller number of students. Yet, there are models in which faculty working with only one to three students on research can still receive teaching credit for their mentoring. In one such model, a group of faculty doing undergraduate research can combine their efforts and form one research seminar course per term. Math majors doing research sign up for this course as a major elective. One of the faculty members whose research students are in the course is designated as the instructor, teaching it as part of their course load that term. In the next term, the course can be offered again with a different faculty mentor listed as the instructor. This approach does require keeping records of which faculty members have mentored students in the past and who has been listed as the instructor for the course.

Also, there needs to be a discussion about how many times a student can earn credit for such an elective course in the major. In the end, however, it does offer faculty more compensation than the fractional model.

No matter what your own motivations are for engaging in research with your students, you will enjoy greater job satisfaction and your students will certainly gain valuable problem-solving experience to take with them into the work world.

1.4 A brief history of undergraduate research in mathematics

Before we move on to the nuts and bolts of leading a team, we'd be remiss if we didn't point out that undergraduate research in mathematics is not a new idea. There has been federal funding available for such projects since at least the 1960s. If you're interested in a more detailed exposition of this history, we refer you to Joe Gallian's thorough summary found in [**78**]. However, we'll give a brief summary here.

Although the National Science Foundation (NSF) has supported undergraduate participants contributing to a professor's broad research program since its inception, the first program aimed directly at undergraduate researchers was the Undergraduate Research Participation (URP) program, which began in late fall 1958 [**148**]. Notably, the Council on Undergraduate Research (CUR) was an instrumental advocate, helping bring this program to fruition. A mere 10 years into the program, a preliminary survey indicated that URP participants were more likely than their peers to be a coauthor on a scholarly paper and more interested in pursuing a Ph.D. to become a professor [**8**]. The URP program supported many students—mostly in physics, biology, and chemistry—throughout its lifespan, but it was cut as a result of federal budget cuts in 1981. Fortunately, this lack of funding did not last long. There was widespread concern in the early 1980s about the sharp decrease in American students graduating with college degrees in STEM (science, technology, engineering, and mathematics) disciplines, so five years later, NSF resumed formal funding of undergraduate research through the new Research Experiences for Undergraduates (REU) program. REU sites were encouraged to assist in efforts to increase populations of women, Native Americans, minorities, and other under-represented groups working in the STEM fields. In 1987, the first eight mathematics REU sites were funded. REU funding continues to this day, supporting as many as 60 mathematics REU sites each year, as of this writing.

NSF REUs are not the only funded form of undergraduate research. In 2006, another NSF grant funded the founding of the Center for Undergraduate Research in Mathematics (CURM), headquartered at Brigham Young University [**51**]. CURM provides support and training to faculty so that they can successfully mentor small teams of two to four undergraduate students at their home institutions. Unlike REU programs, which typically recruit students for intensive full-time summer projects, CURM supports projects during the academic year. CURM accomplishes its work via mini-grants that provide student and faculty stipends as well as travel support so that faculty mentors can engage in professional development workshops and students can present their work. The program not only engages students and encourages them to consider graduate school and STEM careers, but it also provides critical support for pre-tenure faculty, helping them to navigate the challenges of early career work. In 2015, the American Mathematical Society (AMS) selected CURM for its "Mathematics Programs that Make a Difference" award, citing the program's significant efforts to encourage students from underrepresented groups to continue in the study of mathematics [**106**]. Similarly, the Preparation for Industrial Careers in Mathematical Sciences (PIC Math) program has trained faculty and students since 2014, empowering them to work on research problems with business, industry, and government partners [**177**]. Following the lead of these national programs, a growing number of colleges and universities offer internal funding to support joint projects between faculty and undergraduate students.

The rapid growth in mathematics undergraduate research today may make you wonder why it took so long for undergraduate research to become a trend in the mathematics community, especially compared to other STEM disciplines. The first conference about undergraduate research in mathematics took place at Carleton College in 1961 [**134**]. The conference proceedings indicate that undergraduate research was seen as beneficial, but as an activity of lower caliber than the research of practicing mathematicians. Indeed, as late as the 1980s, then-President of the MAA Lynn Steen reported to the National Science Board [**204**]:

> Typically, good undergraduates glimpse the frontiers of science from association with faculty research projects. However, research in mathematics is not like research in the laboratory sciences. Whereas undergraduates can become apprentice scientists in chemistry research laboratories, research in mathematics is so far removed from the undergraduate curriculum that little if any immediate benefit to the undergraduate program ever trickles down from faculty research. As a general rule, undergraduates can neither participate in nor even understand the research activity of their mathematics professors.

Some still hold this view that undergraduates cannot participate in meaningful research, but it is not difficult these days to counter this perspective with an overwhelming body of evidence of quality student work. Since 1987, the Mathematical Association of America has sponsored contributed paper sessions for undergraduates at its annual summer meeting. Since 1991, an undergraduate poster session has been held at the Joint Mathematics Meetings. Both of these sessions have grown in popularity over time with hundreds of students' work represented each year. There are even many regular conferences held specifically for undergraduates to present their mathematics research. Moreover, several journals exclusively publish research conducted with undergraduates in math. This is not to say that student work is always presented in these specialized venues. Gallian [**77**] notes that sole-authored papers by undergraduates appear in such top journals as *Annals of Mathematics*, *Annals of Probability*, *Journal of Algebra*, *Journal of Number Theory*, *Proceedings of the American Mathematical Society*, and more. Since 1995, the AMS, the MAA, and the Society of Industrial and Applied Mathematics (SIAM) have presented the Morgan Prize [**141**] annually to an undergraduate student for outstanding research in mathematics, recognizing the high quality of their published work. In 2015, Gallian [**79**] compiled an overview of the career paths of the first twenty Morgan prize winners; their trajectories indicate that undergraduate research was a first step in a series of significant mathematical achievements.

As student output has increased, resources for faculty interested in undergraduate research have likewise expanded. In 1990, CUR established a Mathematics and Computer Science division that remains active today. The MAA has supported undergraduate research via a committee on research by undergraduates (now a subcommittee of the Committee on the Undergraduate Program in Mathematics) and by offering a mini-course on undergraduate research at the Joint Mathematics Meetings. Today, there is a Special Interest Group of the MAA (SIGMAA) on Undergraduate Research whose purpose is to "broaden participation in undergraduate research in mathematics by supporting faculty serving as mentors and by supporting interested students" [**130**].

We have also noticed a generational change in faculty interest in mentoring students in research; specifically, early-career faculty seem to have a greater dedication to involving undergraduates in research. Part of this may be due to the influence of Project NExT on STEM culture at liberal arts colleges. Begun in 1994, Project NExT selects a cohort of early-career faculty each year from a national pool and "addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, finding exciting and interesting service opportunities, and participating in professional activities." NExT has been influential in promoting undergraduate research, partly by offering a popular annual course on undergraduate research. The course was offered for many years by Aparna Higgins, former Project NExT Director, underscoring the commitment of Project NExT leadership to this practice.

Finally, faculty involved in undergraduate research have participated in a series of impactful conferences. In 1999, the American Mathematical Society (AMS) brought together mathematicians from across the country who had worked with undergraduates in summer research programs [**74**]. A second conference was jointly hosted by the National Security Agency and the AMS in 2006 on Promoting Undergraduate Research in Mathematics [**76**]. With tremendous growth in undergraduate research programs, a third conference was held in 2012, funded jointly by the NSA and NSF on Trends in Undergraduate Research in the Mathematical Sciences (TURMS) [**143**]. Most recently, Mount Holyoke College and the NSF sponsored a mini-conference on New Directions for Mathematics Research Experiences for Undergraduates in 2013 [**175**].

Compared to just a few decades ago, the culture change is profound both for faculty and for students. One survey estimated that about 4,500 undergraduate students were engaged in research in mathematics during the 2010–2011 academic year [**47**]. Although there has been no systematic assessment of the impact of undergraduate research programs, O'Shea argues that undergraduate research may play a role in improving the quality of mathematics education available to students at all types of educational institutions [**171**]. There is room to more deeply understand the impacts of undergraduate research throughout the mathematics community, but one thing is certain. Undergraduate research is a trend that is here to stay!

1.5 Getting started

Undergraduate research in mathematics has a rich history. Are you eager to be part of it? We're happy to provide the resources. In Chapter 2, you will find some general guidance to help you get started. It can be read independently, but it hopefully whets your appetite for more. In the rest of the book, we'll go into detail on various topics, including choosing good problems (Chapter 3), finding students and managing group dynamics (Chapter 4), helping students communicate their results (Chapter 5), finding funding (Chapter 6), running an REU program (Chapter 7), and assessment (Chapter 8). Finally, we'll wrap up with a discussion of future directions for undergraduate research in Chapter 9. Each of these chapters can be read relatively independently of the others, so if you're already working with students and are looking for advice on a particular issue, by all means, jump to the topic that is most relevant to you.

Collaborative research with undergraduate students is one of the highlights of our work. It is indeed *work*, but the benefits to both students and faculty are tremendous, and there is a growing network of supportive practitioners to help you get started. For now, read on as we discuss the finer details of leading an undergraduate research team.

2

A beginner's guide to mentoring undergraduates in research

Note: This chapter is an adaptation of a paper that the authors wrote for PRIMUS [**54**].

2.1 The six fundamental steps

Doing research with undergraduates can be one of the most satisfying aspects of our jobs as math professors. For those of us who love both teaching and doing research, what could be better than mentoring and collaborating with students? It should go without saying that the three authors of this book feel that working with undergraduates on research is incredibly rewarding. We've learned through our collective experience mentoring over 100 students in research that there are several things to consider, whether you are just starting your first project with a student, having only worked with graduate students or peers on research until now, or you already have some experience with undergraduate research. To help you prepare for your next research experience with students, we will discuss the Six Fundamental Steps needed to successfully mentor students in research.

Step 1: Picking an appropriate research problem

Step 2: Recruiting and selecting students to mentor

Step 3: Setting expectations and dealing with group dynamics

Step 4: Starting the research and moving it forward

Step 5: Helping students develop communication skills

Step 6: Preparing for the future

In this chapter we will give an overview of these steps, but they will be explored in greater detail in subsequent chapters.

Before we discuss these Six Fundamental Steps in depth, it makes sense to clarify what we mean both by undergraduate research and success. In Chapter 1, we gave an informal, working definition of undergraduate research. Formally, the Council on Undergraduate Research (CUR) defines undergraduate research as "an inquiry or investigation conducted by an undergraduate student that makes an original intellectual or creative contribution to the discipline" [**28**]. We, the authors, agree that undergraduate research involves students working on problems that no one has solved before, although some people think of undergraduate research simply as students working on problems that the students have never solved before. One reason that we prefer the CUR definition is that, in talking with recruiters from over 50 companies that hire math majors, we have heard them say many times that they are looking for students who have worked for an extended period of time on an unsolved problem (e.g., in an undergraduate research experience or an internship). Companies look to hire students who have had this type of experience because these unsolved problems are precisely the types of problems their employees work on [**52**]. On the other hand, while undergraduate research is similar in many respects to the kind of research you might do with peers, it is important to understand that there are often differences in the way these two sorts of research are conducted. People who claim that you cannot do undergraduate research in mathematics usually have not yet come to this understanding. These distinctions will become clear in the discussion that follows, but for additional references detailing the unique benefits of undergraduate research, see [**52**].

We would also like to recognize that what each of us would consider a "successful" undergraduate research experience may be different. So we encourage you to ask yourself: "What is the reason I am doing undergraduate research?" and "What expectations do I have of my students and of the experience as a whole?" Your answers to these questions will help you determine what success looks like for you and will influence how you approach the Six Steps. Do you want to mentor undergraduate students to further your own research? If so, do you expect to get a publication from mentoring these students, and what type of journal are you aiming for? Do you want to do undergraduate research because it is one of the best methods for teaching mathematical concepts? Do you want to use it as a way to help prepare students for graduate school or non-academic careers? Before we move on, we should be clear that these are all perfectly good reasons for doing undergraduate research, contributing to valid definitions of success.

Let's see how your answers to these questions could affect your approach to mentoring. Suppose you plan to do undergraduate research as a way to strengthen your own research record. This would naturally influence the types of research problems you would choose for your students. You would still need to pick research problems that are appropriate for your students' level of expertise, of course, but these problems would likely be related to your own research program. Students could create examples demonstrating a theorem you have proved or they could take a theorem you have proved and try to apply it to another situation. Let's consider an example of Michael's. In complex analysis, he once proved a theorem that involved operating the function $f(z) = z$ on a family of analytic functions. The function $F(z) = (z - \bar{a})/(1 - \bar{a}z)$ where $a \in |z| < 1$ is a generalization of the function $f(z) = z$ (i.e., $f = F$ when $a = 0$).

Michael had some students formulate an equivalent theorem using F instead of f. He then had them use the pattern of the original theorem with f to prove the equivalent theorem for F.

In addition to the types of problems you choose to work on, the selection of the students to mentor would be influenced by your goal of working on a problem related to your own research. You would probably first look for more advanced undergraduates who may have already taken a first course in your research area such as algebra, number theory, topology, or complex analysis. On the other hand, suppose your decision to do undergraduate research is inspired by a desire to prepare students for graduate school by giving them a taste of what math research is like. This would open up more possibilities for research topics. For instance, Allison enjoys working with students to study games that can be played on knot diagrams. This kind of research is fun and requires little background knowledge on the students' part. She invited several of her research students to invent a new game to be played on knot diagrams and asked them to think about winning strategies for certain starting positions. The goal wasn't to get a published paper, but to give the students experience with asking questions and developing a plan to answer their questions. Some answers required students to learn some game theory, some questions required knot theory to answer, and some questions could be answered by writing a computer program. In this general scenario, questions might be accessible to less advanced students. This type of research project can provide a great opportunity to work with students who are uncertain about continuing in mathematics or to encourage strong, but less confident students.

As Gillman and Szaniszlo state in their description of academic-year undergraduate research at Valparaiso University [**82**], the goals of a successful undergraduate research experience are that the project "models the research experience of mathematicians, provides a growth experience appropriate for the maturity level of the participating student, helps students build meaningful connections to the faculty and the department, and introduces students early in their studies to the discovery of new mathematics." A successful undergraduate research experience focuses on student growth and allows the student(s) to interact with material at a level that goes beyond what they do in the classroom; it allows them to develop their own conjectures and proofs.

Keeping these ideas in mind, let's discuss the Six Fundamental Steps needed to successfully mentor students in research. Aspects of these steps are also discussed in articles such as [**11, 42, 48, 58, 82, 93, 119, 136, 185**].

2.2 Step 1: Picking an appropriate research problem

A common difficulty for faculty who are just starting to mentor undergraduate students in research is to choose an appropriate research problem. Your choice of problem should be informed both by your goals for the project and by the students' background. A problem that fits into your current research agenda will look different than a problem chosen with the main goal of leaving room for students to make their own conjectures. Regardless of your personal goals for the research, it is important to consider the level of the students' background when choosing a problem. It is typical for beginning faculty mentors to pick a research problem that is too difficult for students to finish. How can you avoid this pitfall, especially if you don't have a lot of experience in working with undergraduates on research? Kathryn Leonard (Professor, Occidental College) talks about how she chooses an appropriate research problem by thinking of it as a problem she could solve on a lazy afternoon.

Some qualities of an appropriate research problem for students to work on are: (1) the problem should require a limited amount of background material, (2) it should be specific and concrete, (3) it should have multiple layers starting out somewhat simple and then progressing into higher levels of difficulty, (4) it should be of interest to you and your research community, (5) it should lend itself to creating specific examples and may include using computers, and (6) you should have some idea of how the problem might be solved. A problem with multiple layers is the opposite of an all-or-nothing problem in which the students have to prove a result, and if they don't, then they have nothing to show for their efforts. A recent article about PRIMES (Program for Research In Mathematics, Engineering, and Science), a mathematics research program at MIT for high school students, also describes many of these components for choosing an appropriate research problem [**66**].

As a basic example, consider the following problem. The area of a circle of radius r is $A(r) = \pi r^2$ and the perimeter is $P(r) = 2\pi r$. So, $dA/dr = P$. For what other geometric shapes does this derivative equation hold? If we look at a square with side length x, then $A(x) = x^2$ and $P(x) = 4x$, and $dA/dx \neq P$. However, if we inscribe a circle of radius r into the square, then we can write $A(r) = (2r)^2$ and $P(r) = 8r$ and $dA/dr = P$. What other geometric shapes satisfy $dA/dr = P$ and what relationship does r have in these shapes? Some results are known about what other geometric shapes satisfy $dA/dr = P$, but there are more problems to explore (see [**53**]).

If you can't think of a research problem from your own field that is appropriate for undergraduate students, there are various resources to help you find a problem. These resources include journals that publish undergraduate-level work with easily extendable results such as *Involve*, *SIAM Undergraduate Research Online*, *MACE Journal*, or *UMAP Journal*. Conference sessions in which students present their research, such as at the MAA's MathFest, the Joint Mathematics Meetings, or local MAA sectional meetings, can also be sources of inspiration. If you read an interesting paper or see a talk on work done by undergraduates, you can reach out to the project's research mentor to find out which questions related to their project have yet to be solved and which of those they don't expect to work on themselves. In addition, you may consider online resources such as openproblemgarden.org, which lists problems and recommends certain problems as suitable for undergraduates.

Finally, realize that a little creativity on your part may pay off. Maybe the question you ultimately explore with students is not one you found verbatim from one of these resources, but a problem on which you put your own twist. For example, Lara once attended a conference talk on pattern avoidance in graph-theoretic trees. The speaker's definition was that tree T contains tree t as a pattern if t is a subgraph of T. Lara proposed an alternate definition of tree pattern involving edge contractions. The speaker had not thought of this definition and encouraged her to pursue it. That modified definition fueled two successful summer REU projects where her students took ownership of the new definition, and their work led to cited publications in recognized combinatorics journals.

To read in more depth about these and other ideas for picking good research problems, see Chapter 3.

2.3 Step 2: Recruiting and selecting students to mentor

Recruiting and selecting students to work with you is Step 2. Many students do not realize there are undergraduate research opportunities, or if they do, many assume they are not qualified. Alayont, et al. discuss how to combat these perceptions in [**2**]. Since students may not know to come to you for a research opportunity, you should go to them! There are various ways to recruit students. You can find students in classes you have taught, ask colleagues to recommend students, or send out an email to all the math majors asking for students interested in doing a research project with you. Some students may be naturally drawn to you because they were in a class you taught and enjoyed it, while others may be intrigued by a problem you describe in an advertisement email.

Have you ever had a student come to see you after class to discuss one of the more challenging, unassigned problems at the end of a chapter in your textbook? More generally, can you recall a time when a student demonstrated their curiosity and interest in understanding a class concept more deeply? Students who demonstrate this kind of curiosity are often students who would make excellent undergraduate researchers. One way to identify these students might be to suggest a few optional, challenging homework problems in your classes each week and see who shows an interest in attempting them. When they come to see you to discuss their attempted solutions, you can mention that you are looking for a few undergraduates to work with on a research project and let them know that you think they'd be a great contributor to the team.

Outside of her classes, one of Allison's favorite methods for recruiting students is by giving talks in her university's Undergraduate Mathematics Colloquium. A few years ago, she gave a talk on pseudodiagrams of knots and knot games to students in her department. Afterwards, three students immediately asked her if they could work with her on research. This led to a two-year-long project with these three students that didn't result in a publication, but taught the students valuable research skills and helped them figure out whether or not they wanted to go to graduate school.

Michael has found success with giving a short description or presentation of his research that he can give to students to capture their interest. Michael's research is in complex-valued harmonic mappings that are a generalization of analytic functions, and he studies properties that are preserved under convolutions. When he tries to recruit students to do research, do you think that's how he explains his research? Of course not! Instead, he talks about creating soap films by dipping wire frames into a soap solution. He explains to students that he studies the properties of these soap films. In fact, almost every semester he will bring in a bucket of soap solution with some wire frames one day during class and demonstrate some soap films. Students are enthralled and it is common for a student to contact him later asking him about doing undergraduate research. (You may be wondering, what is the connection between Michael's research and dipping wire frames in soap solutions? The answer is that under certain conditions, complex-valued harmonic functions lift to minimal surfaces, and minimal surfaces can be modeled by soap films.)

Current research students can be the most effective recruiters for potential new students for a research group. On one occasion, Allison and her research student Christopher went to a baseball game with several other faculty and students from their math department. They gave a younger student named Colin a ride to the game, so they all had some time to chat. Christopher spent most of the ride explaining his research to Colin. A few weeks later, Colin showed up in Allison's office asking if he could join the research group. Christopher and Colin were a great research team for the next year. They gave a fascinating talk on their results at a local MAA sectional meeting at the end of the school year, and Colin continued working on a related research problem for a second year with Allison after Christopher graduated.

Most professors start doing undergraduate research by working with one student on one problem. Over time, though, many discover that it's more efficient to have a research group of 2-5 students working on the same problem at the same time. The three of us generally try to set up our research groups so that we have some advanced students and some beginning students. The advanced students are often students who have worked with us longer on research and who have more math background. By having a group with different levels of experience, you can have the more advanced students help you mentor the beginning students in some of the basic ideas and procedures for your research. This mentoring experience is great for the advanced students, and it takes less of your time—this has really helped us to continue to do undergraduate research even as the demands on our time have increased. It's a sustainable model. When the advanced students graduate, beginning students move up to the role of advanced student, and they can recruit new students to become the beginning students. Your own research students know what the research entails and usually have a better grasp than you do on the suitability and compatibility of their peers. Also, your research students will typically recruit students they get along with, so you'll have fewer issues with group dynamics to sort out.

Once you have a system for recruiting students, you should decide how to select students to actually do research. Often, faculty members want students to have a minimum amount of mathematical background. If you are doing research in mathematical biology you might require students to have a differential equations course under their belts. If your research involves proving theorems, you might want your students to have an Introduction to Proofs course. We have found that the most important quality for an undergraduate researcher to have is that she/he is a hard worker who is eager to do research. This is more important, in our experience, than intelligence or level of mathematical background.

Since Michael's research area is complex analysis, he used to only consider mentoring students who had earned an A in an undergraduate complex analysis course. He thought that they needed this foundation just to begin. One semester, he was continuing research with three undergraduates who had begun working with him the previous semester. Unexpectedly, a new, bright-eyed student came to him asking to be in the research group. This student was a sophomore and had not had complex analysis, but he was a hard worker. Michael took a chance and accepted him. He was pleasantly surprised to see both how much this sophomore learned from the more advanced students in the research group and how much he was able to learn on his own.

Allison ran an REU for students where the minimum requirement was just Calculus III. For projects in fields such as knot theory, combinatorics, and statistics, you don't need every single person in the group to have more mathematical maturity than a few semesters of calculus would provide. If your students are motivated, they can fill in the gaps in their knowledge as they are doing research, possibly with the help of their peer collaborators. What's more, when students learn advanced material because they need it to solve a research problem, rather than simply to pass a course, they learn the material much more deeply.

In Chapter 4, we explore additional issues related to finding and selecting students to work with on research.

2.4 Step 3: Setting expectations and dealing with group dynamics

Before getting involved in research, we recommend discussing some ground rules and expectations with your students. Remember that they are likely new to research. There may be norms that you have when working with your peers that undergraduates have not learned or thought about. You probably have a vision of how the project will be organized, but chances are that the students will have a different or hazy vision of how the group should operate. You should sit down with your students and discuss the structure or ground rules for working with you and each other. Some items to discuss are clear, such as when and where you will meet for research. Other items to discuss include the students' weekly time commitment to the project and what the students should do if they cannot make it to a meeting. Also, it is important to discuss some long-term expectations, such as what end products they will be expected to produce (e.g., a final written report, a presentation to people outside of the research group, a poster).

Another area of discussion that is important but is often not discussed is group dynamics. By group dynamics, we mean how the individual members of the research group interact with each other. Besides differences in mathematical background, students in your research group have differences in ways they communicate with each other and deal with situations. Kathryn Leonard [**119**] mentions that some of the group dynamics among the students include lack of communication, sporadic involvement, power struggles, and closure to constructive criticism. In [**93**], Hannah Callender Highlander talks about emailing her students before they began researching and asking them to respond to a set of questions such as "In past group experiences, what has worked for you and which hasn't?" "What is the best way for you to receive criticism?" "If you start to struggle, what is your plan?" "What are your current questions or concerns?" During the group's first meeting, Hannah leads a conversation about the students' various responses, discusses effective ways to handle conflict and deal with criticism, and talks about each member's different expectations. She writes, "As one student opens up, it often provides relief for other students who are feeling the same way but are too afraid to admit it." Setting expectations for the project is a critical teaching moment that can help foster an environment where the students are comfortable making contributions and asking for guidance as needed, rather than shutting down when they are challenged.

In Chapter 4, we discuss setting expectations and managing group dynamics in more detail. We also address issues that can arise in research groups as well as things to consider when mentoring students from underrepresented minority groups.

2.5 Step 4: Starting the research and moving it forward

Remember, you have spent a lot of time and effort learning how to do research. Perhaps you've forgotten what it was like when you first started. If you think back to the worries you had in the beginning of your research career, you'll understand that it is an important investment in your students' psyche to discuss the process of doing research with them, especially as they are just beginning. During the research project, it is useful for students to be reminded that it is the rule, rather than the exception, that researchers struggle.

The CURM program [**24**] runs a faculty training three-day workshop every summer. During the workshop, one of the activities is to create a list of key things that faculty know about research that their students probably don't know. Below is a list of the 12 most common key items. This list is great to share with undergraduate students as they do research. Doing this helps the students learn that feelings of frustration and slow progress are not due to their own lack of mathematical skills, but such feelings are common to all mathematicians doing research. The list includes:

- Don't be afraid to ask: "Why?"
- It's OK if you don't understand an idea the first time (or the second time, or the third time).
- We all get stuck and frustrated. When this happens,
  - take a break,
  - explain to someone (mathematically trained or not) why you are stuck,
  - review background material,
  - see if the problem can be modified (problems are not set in stone),
  - check hypotheses or assumptions,
  - work out a simple example, and
  - keep going!
- Published work is not always correct—including the work of your faculty mentor.
- Be open to different ideas and approaches for solving a problem.
- Your project might go in a completely different direction than you think it will.
- Everything takes longer than you think it will. Be patient.
- It's OK to make mistakes. Making mistakes is a great way to learn!
- Hard work and perseverance are necessary (but not sufficient). In fact, hard work is the most important feature of a successful student.
- You don't need to know (and cannot know) all the background.
- Learn to collaborate.
- Research is challenging, but rewarding.

In starting to actually do research, it is a good idea to begin by presenting some background material. There are various ways to do this. Some professors prepare notes for students to learn from, some professors have students read material from books or journals that are written at the students' level, and some professors have students read faculty-level research papers. If you choose this last approach, choose the faculty-level research papers judiciously. Once, a colleague came up to Michael and said that he had tried to get an undergraduate student to do research, but he was unsuccessful. Michael's colleague mentioned that he gave a student in his real analysis class his latest research paper in functional analysis. He told the student to read it and drop by when he had questions. The student never came by. Of course, the problem was that the colleague's paper was far too advanced for the student to read. (It probably would have been difficult for most of us to understand it!) So, make sure the material you give to students is accessible to them. Knowing what level material is digestible by students often takes time and experience.

Another mistake that faculty often make in getting students started with research is at the other extreme. Mentors think that they need to explain a tremendous amount of background material before students can begin working on research. This is the paradigm that many of us mentors were exposed to when we began researching in graduate school. Unfortunately, this kind of thinking is one reason some faculty mistakenly believe that undergraduate students cannot do research. These doubtful professors think of undergraduate students doing research in the same way they think of Ph.D. students doing research: the faculty must present a lot of mathematics before the students can start looking at research problems. However, we have found that if undergraduate students focus on just a few essential ideas that they need to start researching, they will pick up many of the other important ideas as they become involved in the work. Moreover, it will be more interesting and enjoyable for them to work on research right away.

Michael once read a colleague's REU proposal in which the colleague wanted to spend the first five or six weeks of the eight-week REU teaching background material to the undergraduate students and then have them work on research during the remaining few weeks. Such an REU proposal would not get funded; the time periods for learning new material and working on research problems were precisely backwards.

We advise you to avoid the common mistake of thinking you have to explain a lot of material before students can begin to start working on research problems. So how much time is appropriate for background material? Early on, Michael would present background material during the first 15-25% of the research project time period. During this time, students would work on introductory problems while learning about potential research problems. Next, he would summarize the potential research problems that they had discussed and let the students choose the problems they wanted to work on. After following this approach for several years, Michael decided to write up notes on the background material and included a few hundred exercises and exploratory problems for the students to do. These notes became two chapters in a book on topics in complex analysis that undergraduate students could explore [**19**].

Even if you don't intend to write a book, focus on giving students a clear place to start their work. Lara's research is in enumerative combinatorics. She likes to give students a handful of key definitions at the first meeting. The students discuss those definitions until all group members can give appropriate examples to go with them. Then, she gives students two or three possible open questions related to those definitions and encourages the students to explore and express a preference for which question to start with at the second meeting. The students have true ownership of their problem from week one, rather than much later in the project.

Now, what happens when you start meeting with students who are researching? Our experience is that students appreciate having a structure or a plan for the meetings. When any of us teaches, we prepare what we want to do during the class. When we conduct a committee meeting, we prepare an agenda. When we mentor students, we should also prepare a structured plan. Of course, we can deviate from the plan if a juicy idea comes up that needs some further prodding. We find it is easier to deviate from the plan if we have prepared beforehand.

During the first few meetings with new students, we have them read some material and work on specific problems related to the material. We ask students about the reading and ask them to discuss their solutions to the assigned problems. If they're working in a group, we encourage students to work together on the problems. Just as in a class, if one student cannot solve a problem, it's likely that others in the group also need help on it. In that case, we can spend some time discussing that problem as a group. As the students complete the background material phase, the structure of the meeting evolves. In later meetings, we will often review some of the ideas that we have discussed, list specific items that could be worked on, and give an idea of what it would take to work on each item.

For example, suppose the group has been exploring a theorem and, based upon modifying parts of the theorem, we have come up with a conjecture. At this point, it's good to recognize that we have several tasks we could explore. First, someone could write up the notes from the day's discussion. Second, someone could do a MathSciNet, arXiv, or internet search to see if another researcher has written a paper related to this conjecture. Third, someone could use a computer to try to generate more examples of this conjecture and see if we can find a counterexample. Fourth, in trying to prove the conjecture, it would be useful to understand the essential ideas behind the proof of the original theorem. Someone in the group might reread the proof and present to the group the major components of the proof. Fifth, someone could try to prove the conjecture by mimicking the proof of the theorem and inserting the appropriate new components from the conjecture into the parts of the proof where the old components of the theorem are. The students can decide who would like to work on which tasks. Each student picks at least one task, and more than one student may work on the same task. Finally, the students should know that during the next meeting, everyone will be expected to report on the task they have been working on. Typically, most of the tasks have not been completed by the next meeting, but the students should still give a progress report and share any unexpected difficulties. Then, the team will choose new tasks or continue to work on the previous tasks. Students are given some guidance on how to move forward; they are not alone. On the other hand, they have the freedom to choose assignments that match their abilities and skills. Also, it is helpful for students to have something specific to do before the next meeting.

During the academic year, we usually schedule one or two one-hour meetings per week. The nature of the meetings encourages everyone to talk and share ideas. Students see how research is done in mathematics, they learn problem solving techniques, and they develop independence. Some of these ideas are discussed further in [**11**].

We'd be remiss if we neglected to mention that you should to talk with your students and really get to know them. That's one of the perks of doing this kind of work: building stronger mentoring relationships with students. Occasionally, you can brighten your students' moods and break up the research routine by bringing treats (e.g., donuts or pizza). During summer research programs, you can host game nights, movie nights, or picnics. You can take your students to a baseball game or head to the mountains together for a hike. If you have a sense that your students are working extra hard or are dragging, or if you have something to celebrate, take an impromptu trip to get ice cream!

2.6 Step 5: Helping students develop communication skills

Another key difference you'll find in working with undergraduates on research is that they need guidance in writing and presenting mathematics. Developing these communication skills is incredibly important. It requires practice on the student's part and helpful feedback on the professor's part. Recruiters from over 50 companies that hire mathematics students have suggested several skills that mathematics students should develop in addition to learning about mathematics. Communication skills (i.e., writing and speaking) is one of these. This has changed the way Michael emphasizes communication when working with research students. Now, he requires each undergraduate student to give a talk outside of the mathematics department, and each research group must write a research paper. These two activities often have a significant impact on the students. Giving a presentation and writing a paper also brings closure to a project and provides the students with two tangible end products of the research experience that they can put on their résumé or curriculum vitae. Both activities can also be helpful resources for your next research group; when the next group of students makes slides for their presentation, they can use the previous research group's presentation slides as an example of what a talk should look like. The research paper provides some background material written at an appropriate level for your new students to read as they begin their research. Many students (like many professors!) have a mild to intense fear of standing in front of a group of people to talk about their research. The antidote to this fear is practice. The more often students practice giving talks, the more skilled they become at presenting, and the more confidence they develop in their communication abilities. So have students give regular presentations to the group about their research. During an 8-week REU, for instance, you can have students give presentations once a week. After a presentation, the audience members can provide feedback, like: "I did not understand what you meant when you said..." or "I think an example or a picture would help people understand what you mean." Outstanding detailed advice on presenting posters and presentations can be found in Higgins et al. [**92**].

When students write a paper about their research, make sure they start early. One of the most common problems we have heard is that the professor and the students wait too long before beginning to do the writing. As a result, they lose steam and the work never gets written up, or they find themselves in a mad dash to write a paper in a matter of days. To combat this, we have found it helpful to have the students start writing portions of a research report early on (e.g., a first attempt could be done at the midpoint of the project). The students then submit it to us, and we give specific feedback. We ask the students to revise the write-up by incorporating our feedback and including any new material. We recommend that you have your students submit two or three drafts before they get to the last few weeks of the project. Often, the end of a research project is at the end of a semester or at the end of the summer. If the research group waits until the end of the project to start writing the paper, then they will be stressed with all the other obligations at the end of the semester or summer. This will make it hard to complete their task, or at least the final product will be of lower quality. When a research project is over, it is even more difficult to write the paper.

A more proactive approach is to have students write up their work in even smaller pieces than an entire project draft paper. When Lara works with a team of students, she sets up a shared Dropbox folder for the team at the first meeting. Whenever students do computer work, write down a proof, or write up notes from a meeting, the students save their work in this joint repository. In fact, every time a student has an idea for a proof, Lara asks the student to write it up and put it in the Dropbox folder within the week. She returns the draft with comments at the next meeting and takes care of editing on a one-proof-at-a-time basis. This approach has several benefits: writing up one result at a time makes smaller less-intimidating pieces for the students to work with. Also, having a collection of well-polished small documents makes it easier for students to look back at what they’ve accomplished at any point in the project without losing information. Finally, when the students are ready to write a paper, their focus is on writing the narrative between the proofs since the actual mathematics has already been through multiple drafts of editing. In any field, writing is a process that benefits from rewriting and revisions. Starting early with the writing process will help to avoid problems later.

We share more detailed advice on how to help students communicate their results in oral, written, and other forms in Chapter 5.

2.7 Step 6: Preparing for the future

As your project draws to a close, there are some things you can do to help improve your ability to mentor undergraduate research students in the future. First, have your research students write down research problems they would have liked to work on if they had had more time. Michael and Allison have their research students include this at the end of their research paper. This list of problems can be an excellent source of future problems for new research students. This strategy works especially well if you have your new students read the previous students’ research paper as background material. Your new students already have a list of new research problems at the end of the paper they just read.

Second, near the end of the research project, take some time to reflect on your experience. Write down notes on what went well, what didn’t go well, and what you would like to do differently next time. This is important to do while the thoughts are fresh in your mind. These notes will help you to improve in your mentoring skills. For instance, at the end of Allison’s first summer REU, she discovered that some of her students had struggled with self-doubt and the feeling that they needed to prove themselves earlier in the summer. Knowing this, Allison and the other REU mentors made a note to give students more pep talks and openly address this issue at the beginning of their next REU.

Third, keep a record of your mentoring efforts. This includes keeping a spreadsheet with the names and contact information for the students you worked with, the talks they gave, the papers they wrote, any awards they received, and their future plans. You can also write one or two paragraphs about each student to help you write future letters of recommendation. It is easy to remember all these things right after you work with these students, and maintaining a file with students' information becomes useful as you work with more and more students. Information about your students' accomplishments is beneficial for tenure and promotion, for grant proposals and reports, and for teaching and mentoring award nominations.

Finally, think of ways you can share the results of your undergraduate research efforts with your dean, institution, and alumni. In doing so, concentrate more on talking about your students and what they accomplished instead of talking about yourself. Take photos of your students working on research and giving talks (and get their permission to use these photos) so that you can share them with your institution's public relations people to help you promote your work.

Now that we've discussed the Six Fundamental Steps, let's wrap up by summarizing some of the common pitfalls in doing undergraduate research and contrast these pitfalls with productive suggestions. These are:

- DON'T give students a research problem that is too difficult or is an all-or-nothing problem. DO pick a problem inspired by other undergraduate work or that you could solve in a short time period. Also, DO be willing to recalibrate as the project evolves.
- DON'T wait for difficulties to arise. DO discuss expectations with the students early on.
- DON'T try to give too much background material. DO focus on giving ample time to actually work on a research problem.
- DON'T improvise your meetings with students. DO provide a predictable structure that encourages everyone to participate and ask questions.
- DON'T assume that your students know how to give presentations. DO have the students practice giving talks in a safe space before they give talks in public.
- DON'T wait until the end of the research experience to begin writing. DO encourage students to write up drafts of their conjectures and results as they discover them.

We hope this rough guide helps you think more deeply about how to be a superb research mentor, whether you're a novice or a seasoned undergraduate research mentor. If you found what you read here useful, read on! We have many more ideas to share in the coming chapters.

So, to answer the question “Where is undergraduate research in mathematics headed in the future?” let’s consider CUR’s five strategic pillars. In Section 9.2, we focus on Pillar 1 and the notion of integrating research into the curriculum. Section 9.3 offers ideas for meeting Pillar 4’s goals of innovation in undergraduate research. We treat Pillar 5’s idea of internationalization of research in Section 9.4. Pillar 3 was discussed in Chapter 4, but we will further expand on this pillar in Subsection 9.5.2. A discussion of Pillar 2 can be found in Chapter 8.

9.2 Integrating and building undergraduate research into curriculum and coursework

When undergraduate research in mathematics was first being done, it typically involved one faculty member mentoring one mathematics student. This approach provides the student with a lot of individual guidance, and such a setup is a good starting point for a faculty member who is just learning how to mentor students in research. But it is not reasonable to have one faculty member simultaneously mentor many students in this way. As we mentioned in Chapter 2, another approach is to have a faculty member mentor a group of 2-5 mathematics students working on the same problem at the same time. In this model, having some advanced students and some beginning students work together can be quite successful. The advanced students are often students who have worked longer in the research group and who have a stronger math background. By having a group with different levels of experience, you can have the more advanced students help you mentor the beginning students in some of the basic ideas and procedures for your research. This mentoring experience is great for the advanced students, and it takes less of your time. Indeed, this has really helped us to continue to do undergraduate research, even as the demands on our time have increased. In other words, it is a sustainable model. In Chapter 1, we discussed the benefits for students in doing undergraduate research, so it would be great to have as many math majors participate in undergraduate research as possible. While the group model does allow more mathematics students to have an undergraduate research experience, such a model still leaves out many mathematics students. Are there other models for doing undergraduate research that create opportunities for a larger number of mathematics majors?

Yes! One model that is being implemented in some other disciplines (and occasionally in mathematics) is integrating undergraduate research into the curriculum. How can undergraduate research be integrated into collegiate-level mathematics courses and the mathematics curriculum? This is a question that the mathematics community will encounter more and more, and the possible answers to this question are varied, with some unexplored. We offer some initial approaches that are helping to shape the best practices for the future. We will group these into four types:

- a standard course that is augmented with a few mini-research problems,
- a standard course that is augmented with one major research problem,
- a course with a fairly equal mixture of the instructor presenting new material and students working on research problems/projects, and
- a course in which students work only on research problems.

One of the easiest ways to integrate undergraduate research into the curriculum is to augment a standard course with a few mini-research problems. In this scenario, a faculty member teaches a standard course, but includes a few small research-like problems for students to explore either individually or in groups. While these problems may not be "unsolved" research problems, they may be research-like problems that are new to the students and that get the students prepared to solve more traditional research problems in the future. Let's look at an example.

Alicia Prieto Langarica at Youngstown State University (YSU) teaches a Quantitative Reasoning class (i.e., math for liberal arts majors). This is one of her favorite classes to teach. She lectures during half of the class and then either the students work on worksheets (mini-open-problems/real life questions) or they have a class discussion of current events in the news and their relation to quantitative reasoning. In addition, the students work on a group project called: *What would you change at YSU and how?* For this project, the groups must not only propose a change, but they must consider where the money will come from to make the change or calculate when the initial investment will pay for itself. During the last week of class, Alicia invites university stakeholders who can potentially implement the changes the students have suggested. Amazingly, several of the ideas that were generated during the class have actually been implemented. For example, a group studied the possibility of YSU having their own app. The students looked at other universities that have an app, researched how much these apps cost, and explored what services the app provides. A semester after YSU's app was created and implemented, another group of students looked into adding an extra shuttle service during the winter months. They considered how to add the shuttle schedule to the app so students would not have to wait outside in the winter weather for the shuttle.

A second way to integrate undergraduate research into the curriculum is to have students work on one research problem during a standard course. This is similar to the approach mentioned above, except that students work on just one main research problem instead of several mini-problems. Here are some examples.

Anthony Tongen, Brian Walton, and colleagues in the biology department at James Madison University have taught an interdisciplinary math and biology course called *Mathematical Models in Biology* [**123**]. The prerequisites for the course are just one semester of calculus and a course in statistics (which make up the quantitative requirements for biology majors). There are no biology courses required as a prerequisite. Keeping the prerequisites minimal allows for more biology and mathematics students to take the course. The course is a project-based math modeling course. One year, the instructors decided to have an overarching theme for the course, and they implemented the idea of students acting as consultants for a funded research project that "provided a unifying thread, and students could see how aspects of the course would help them address their research questions." Most of class time is spent on students either working on examples that are based upon their pre-class reading or working in teams on their assigned semester-long research project. This research project has multiple benchmarks to guide the students, and all the projects relate to an overarching theme, as if they were part of a large science laboratory.

Another example involving students working on one research problem in a standard course is a second semester abstract algebra course that Tom Sibley at St. John's University teaches. At St. John's, the first semester abstract algebra is a required course for math majors and covers the first half of Gallian's text. The second semester course is an elective taken by math majors planning to attend graduate school. They can cover the second half of the text more quickly, which gives them some extra time in the course to do research. At the beginning of Abstract Algebra 2, Tom gives each student an unsolved research problem (or at least one that is unsolved as far as Tom knows when he gives it). These have included projects that involve investigating "trivial rings" (i.e., abelian groups whose only multiplication is identically zero), "derivative rings" (i.e., rings with an operation satisfying the differentiation rules for sums and products), and "ideal rings" (i.e., rings all of whose subrings are ideals). During the extra time in the course, Tom spends class time presenting some of Gallian's special topics (like wallpaper patterns) while the students can devote their out-of-class time to working on their research problem.

As a third example, Selenne Banuelos at CSU Channel Islands covers the material from Lay's *Linear Algebra and Its Applications* in the first 13 weeks of her linear algebra course. Then, in the last two weeks of the class, students spend their time both inside and outside of class working on a research case study or application project from the textbook. (See [**116**].)

There are some other books that include research or research-like problems that could be used in a standard course to give students experience in working on research. *Keeping it R.E.A.L.: Research Experiences for All Learners* [**127**] contains short-term research problems involving beginning programming and numerical methods. *A First Course in Math Modeling* [**83**] has a list of UMAP Modules (COMAP) that are research-like projects for a math modeling course. *Explorations in Complex Analysis* [**19**] contains additional topics with shorter and longer research problems for an undergraduate complex analysis course. *Introduction to the Mathematics of Computer Graphics* [**23**] starts from the level of Calculus and teaches the basics of linear algebra to bring students up to the level of making small videos and movies. There is a need for more resources such as these that provide research problems for students to work on that are related to a specific course.

A third way to integrate undergraduate research into a course is to mix the presentation of new material with students working on research problems/projects. This is a situation in which about half the time, the instructor is teaching new content material, and the other half of the time, the students are working in groups on research problems. The research problems reinforce the fundamental course content so that the faculty member does not need to directly teach all of the material.

An example of a course like this is the Mathematical and Computational Modeling capstone course taught by Steve Pankavich and Mike Nicholas at the Colorado School of Mines. Steve says that the goal of the course is to "provide upper-divisional undergraduate students with an opportunity to hone their independent critical thinking, presentation, and programming skills, synthesize the knowledge developed within their other applied mathematics coursework, and solve applied problems that contain strongly mathematical and computational foundations. It is not intended to be a lecture-style course, and students work in small groups to analyze and solve real-world problems. A large proportion (appx. 50-70%) of the course grade is based on the successful completion of a group project, in which each group of 3-4 students chooses (or in some cases, is presented with) an applied topic of interest to them. A written report between 15 and 20 pages is due on the last day of class, and during the final week of the semester a mini-symposium, consisting of presentations by the student groups, is held during class with participation faculty from across campus, as well as applied mathematicians and scientists from industry. Because students are typically given the freedom to choose their own research projects within applied mathematics, each instance of this course has featured different group projects, sometimes focused on drastically varying problems, and distinct methodologies are presented to solve these problems." [**172**]. One research problem the students investigated was related to the Ebola epidemic (2013-2016) in western Africa (Liberia, Sierra Leone, and Guinea) that killed more than 8,000 people. The research problem was to determine optimal treatment strategies and site placement to mitigate the spread of Ebola in these countries.

As another example, Alicia Prieto Langarica has taught an Operations Research/ Numerical Analysis class in which she lectured about 75% of the time while the students worked on group research problems the rest of the time. To solve their applied math research problems, the students had to use methods learned in class. One project was to help Youngstown State University library schedule their student workers, while another involved optimizing the scheduling of different machines that a company used in making bottles for various businesses. (The company saved some money. It was not a lot, but it was very exciting that the students' research had a measurable positive impact!)

As a third example, Lara teaches an Experimental Mathematics class at Valparaiso University. Each class meeting, the students are introduced to a new math problem, they write computer code to collect data about the problem, and then the students use the resulting data to make conjectures about the problem's solution. Students build confidence in developing their own mathematical questions and conjectures, as well as in programming skills, by completing the regular coursework. However, one third of students' grades in the class consists of conducting an individual mathematical experiment and reporting on it in writing and in oral presentations periodically throughout the semester. Students have conducted a variety of research projects through this course ranging from number theory to combinatorics to geometry.

A fourth way to integrate undergraduate research into the curriculum is to have a course in which students work just on research problems. Here, the faculty member is not presenting new material to the students. Instead, the students are learning course material through working on the research problems. A major goal of the course is for the students to learn how to solve research problems.

Pamela Harris at Williams College has taught an Undergraduate Research Topics in Representation Theory course following this model. There were seven students enrolled in the course, and they were divided into two groups, each working on an open research problem in the combinatorial representation theory of Lie algebras. At the start of the course, students learned background material by reading a paper Pamela wrote. As the students worked on their research, they were required to write a paper that they revised and resubmitted four times throughout the semester. The final "exam" for the course was a 30-minute oral research presentation on students' research that was open to the department.

A second example of a course in which students only work on research problems is a Mathematical Seminar course that Zhifu Xie taught at the University of Southern Mississippi. This is a one-credit course that students can take while they are working on a research project. The course helps students to think through a possible research topic, understand how to organize and conduct research, and present research results in oral and written formats. All students in this class must conduct a research project in mathematics (pure or applied) while taking this course.

A third example is the Mathematics Practicum course taught by Jill Dietz and Paul Roback (and many other teams) at St. Olaf College. This course takes place during the January interim term, which is a four-week period between semesters during which students only take one course. In this course, students work in groups on separate projects suggested by local companies and organizations and vetted beforehand by the professors. For most of the course, students work intensively in their teams on the problem. Then they give a final presentation to the company and submit a written report, data analysis, software program, etc., depending on the project.

The semester-long PIC Math course follows the model of the Mathematics Practicum. The goal of PIC Math is to prepare undergraduate students for nonacademic careers by having students work in small groups in a class on solving a research problem that comes directly from industry. The course culminates in the student groups giving an oral presentation and writing a final research report. We will say more about the PIC Math program in Section 9.3 on *Innovation and Collaboration in Undergraduate Research*.

Earlier, we mentioned that CUR offers *Institutes* to help faculty members and administrators explore ideas and implement new approaches for doing undergraduate research. One such institute is *Integrating Undergraduate Research into the Curriculum*. This Institute provides a forum to discuss how to engage undergraduate students in research in the classroom, examine existing models, and develop materials for implementing these models in your own classroom. The Institute consists of presentations on issues that often arise in this setting, discussion and brainstorming sessions, and breakout sessions to help participants develop a specific plan for how to integrate undergraduate research into one of their own courses. (See [**33**] for more details.)

We know that there are many benefits for students in doing undergraduate research (see Chapter 1), and it would be great to have as many math majors participate in undergraduate research as possible. The current models of either one-on-one mentoring or a small group being mentored by one faculty member are great, but they still leave out many mathematics students. Integrating undergraduate research into the curriculum is one promising answer to the question of how to help more undergraduate students participate in research. There are disadvantages and advantages to having larger groups of students do research. These are similar to the disadvantages and advantages that we see when we shift from teaching a student one-on-one to teaching a group of 3-5 students to teaching a large class of students. For example, having a larger class doing research projects may result in the projects not being as in-depth, and the interaction between the faculty member and the students is not as personal. On the other hand, more students get to participate in research and receive some of the benefits of doing research. Also, it makes a classroom more active, which has also been shown to result in more student learning, and some of us would say, more excitement. However, we need to devise more and better methods for introducing research into undergraduate mathematics courses.

Where will the future of integrating undergraduate mathematics research into the curriculum lead? No one yet knows, but we are confident that our community of creative academics will come up with some exciting ideas. As with STEM teaching, the mathematics community may learn from seeing what other STEM disciplines are doing. Perhaps some mathematicians will even devise some new approaches to incorporating research into the curriculum. There is a lot of potential for growth in this area.

9.3 Careers for mathematics majors and innovation and collaboration in undergraduate research

Most current mathematics undergraduate research involves research problems devised by faculty members. This makes sense, since professors are the mentors of undergraduate research teams, and they often propose problems that they have an academic interest in working on. However, there are abundant open research problems that mathematicians who are outside of academia work on, and the number of these research problems is rapidly expanding. These problems provide great opportunities for mathematics students, not only because of the sheer number of open, exciting problems, but also because there is a great need for students with strong mathematical abilities to work in business, industry, and government (BIG). Unfortunately, most faculty members are unaware of this opportunity.

For many years, leaders in the BIG community have discussed the shortage of science, technology, engineering, and mathematics (STEM) employees. They have talked about the need for qualified STEM workers in the U.S. workforce. This need is a result of both a decrease in the number of U.S. students earning a STEM degree in college and an increase in the number of available STEM-related careers. In fact, the U.S currently lacks enough qualified STEM-trained workers to fill available jobs. In 2012, the PCAST report described a need for producing one million more college graduates in the STEM fields [**164**]. The U.S. Department of Commerce released a brief in 2011, reporting that “STEM occupations are projected to grow 17.0 percent from 2008 to 2018, compared to 9.8 percent growth for non-STEM occupations” [**112**]. The educational group Change the Equation released its 2012 report calling for more STEM graduates and showing that “for every one person without a job, there were two STEM-related positions available” [**205**]. Similar reports have come from the Association of American Universities [**146**], the Council on Competitiveness [**100**], the National Academy of Sciences [**166–168**] and the National Association of Manufacturers [**165**].

To increase the number of students going into STEM careers, there is a need to better connect the work of BIG and the work of educational institutions. The National Academies of Sciences produced a report in 2012 presenting ten breakthrough actions vital to the U.S.’s prosperity and security. Among their recommendations are that businesses and universities should work closely together to develop new programs that “address strategic workforce gaps for science-based employers” and that BIG should be involved in university programs “to provide internships, student projects, advice on curriculum design, and real-time information on employment opportunities.” While the study focused mostly on university graduate programs, the ideas are still valid for undergraduate students since they become the graduate students of the future [**168**]. Also, the Society for Industrial and Applied Mathematics (SIAM) released the report “Mathematics in Industry” in 2012, containing a list of suggestions and strategies for partnerships between math and industry based on surveys, interviews, and focus group sessions with mathematical and computational scientists from industry [**72**]. A major