

# Contents

<b>Introduction</b>	xi
<b>Acknowledgments</b>	xv
<b>Part 1 Algebra &amp; Geometry</b>	1
<b>1 Computer Algebra Systems and Elements of Algorithmics</b>	3
1.1 Terms and notation	4
1.2 About data types and data structures	5
1.3 Elements of algorithmics and algorithmic problem solving	8
1.4 Glossary	16
<b>2 Topics in Classical Geometry</b>	19
2.1 Review: Matrices, vectors, and lines	19
2.2 Rigid transformations of the plane	25
2.3 Complex numbers in classical geometry	27
2.4 Three centers of a triangle	30
2.5 Glossary	33
<b>3 More Topics in Classical Geometry</b>	35
3.1 Lab 2: The Euler line	35
3.2 The Simson line	37
3.3 Conics	40
3.4 Glossary	45
<b>4 Topics in Elementary Number Theory</b>	47
4.1 Number of primes and the Riemann Hypothesis	49
4.2 Algorithms from elementary number theory	50
4.3 Pythagorean triples	56
4.4 Lab 4: Plotting legs of primitive Pythagorean triples	58
4.5 Linear Diophantine equations in two variables	59
4.6 Lab 5: Industrial application of an LDE in three variables	62
4.7 Glossary	63
<b>5 Topics in Algebra: Solving Univariate Algebraic Equations</b>	65
5.1 Roots of univariate polynomials	66
5.2 Geometry of cubic equations: Counting the number of real roots	69
5.3 Lab 6: Solving cubic equations using Vieta's substitution	74
5.4 Nonnegative univariate polynomials	76
	vii

5.5	Glossary	78
<b>6</b>	<b>Topics in Algebra: Bivariate Systems of Polynomial Equations</b>	<b>79</b>
6.1	Linear systems of two equations	80
6.2	Nonlinear systems of polynomial equations: Motivating example	82
6.3	Solving nonlinear polynomial systems	85
6.4	Implicitization of plane curves	94
6.5	Glossary	96
<b>Part 2</b>	<b>Calculus and Numerics</b>	<b>97</b>
<b>7</b>	<b>Derivatives</b>	<b>99</b>
7.1	Review: Definitions, notation, and terminology	100
7.2	Convexity of a univariate function	104
7.3	Some facts about functions and derivatives	105
7.4	Lab 8: Constructing a square circumscribed about ellipse	110
7.5	Glossary	112
<b>8</b>	<b>Definite Integrals</b>	<b>113</b>
8.1	Review: Some basic concepts and facts of univariate integral calculus	114
8.2	Area of a region bounded by a simple closed curve	116
8.3	Lab 9: Submergence depth of a body of revolution in equilibrium	121
8.4	Solving some ordinary differential equations	123
8.5	Glossary	127
<b>9</b>	<b>Approximating Zeros of Functions by Iteration Methods</b>	<b>129</b>
9.1	Fixed point iteration method	130
9.2	Newton's method	135
9.3	Lab 11: Kepler's Equation and deriving Kepler's Second Law	137
9.4	Lab 12: Exploration of the logistic maps	140
9.5	Glossary	141
<b>10</b>	<b>Polynomial Approximations</b>	<b>143</b>
10.1	Taylor polynomials	145
10.2	Interpolating polynomials in the Lagrange form	147
10.3	Piecewise polynomial interpolation: Splines	150
10.4	Approximating large data sets: Regression	153
10.5	Two real-life applications of the LS method	156
10.6	Glossary	158
<b>11</b>	<b>Trigonometric Approximation</b>	<b>159</b>
11.1	Short review of trigonometric functions	160
11.2	Fourier series	164
11.3	About the accuracy of trigonometric approximations	167
11.4	Celebrated classical application of Fourier series	170
11.5	Glossary	173
<b>12</b>	<b>Fourier Analysis in Music and Signal Processing</b>	<b>175</b>
12.1	Introduction and background	175

Contents	ix
12.2 Fourier series and periodic signals	177
12.3 The Fourier transform for non-periodic signals	180
12.4 The Discrete Fourier Transform	182
12.5 Fourier series in signal processing	187
12.6 Glossary	188
<b>Part 3 Probability and Statistics</b>	<b>191</b>
<b>13 Probability and Statistics Basics</b>	<b>193</b>
13.1 Review: Some basic concepts of probability	194
13.2 Some discrete probability distributions	197
13.3 About continuous probability distributions	201
13.4 Law of Large Numbers	204
13.5 Central Limit Theorem	206
13.6 Glossary	208
<b>14 Computer Simulation of Statistical Sampling</b>	<b>209</b>
14.1 Random number generation	209
14.2 Lab 17: CLT and LLN in action: Life expectancy in the world population	211
14.3 Sampling from non-uniform distributions (optional)	213
14.4 Monte Carlo methods for finding integrals and areas	215
14.5 Lab 18: Buffon's needle problem	222
14.6 Glossary	223
<b>15 Simple Random Walks</b>	<b>225</b>
15.1 Simple random walks on integers	226
15.2 Lab 19: The gambler's ruin problem	231
15.3 Random walk on the square lattice	233
15.4 Lab 20: Drunken sailor problem	234
15.5 Glossary	235
<b>A Data for Lab 17 in Chapter 14</b>	<b>237</b>
<b>Bibliography</b>	<b>239</b>
<b>Index</b>	<b>241</b>