

Contents

Preface	xi
Introduction	1
Chapter 1. Summary of background material	7
1.1. Flatness	7
1.2. Morphisms locally of finite presentation	9
1.3. Étale and smooth morphisms	13
1.4. Schemes as functors	23
1.5. Hilbert and Quot schemes	29
1.6. Exercises	30
Chapter 2. Grothendieck topologies and sites	35
2.1. Sites	35
2.2. Presheaves and sheaves	38
2.3. Cohomology of sheaves	50
2.4. Simplicial topoi	55
2.5. Exercises	64
Chapter 3. Fibered categories	69
3.1. Definition of fibered category and basic properties	70
3.2. The 2-Yoneda lemma	74
3.3. Splittings of fibered categories	77
3.4. Categories fibered in groupoids	78
3.5. Exercises	84
Chapter 4. Descent and the stack condition	87
4.1. Faithfully flat descent	88
4.2. Generalities on descent	93
4.3. Descent for quasi-coherent sheaves	98
4.4. Examples	103
4.5. Application: Torsors and principal homogenous spaces	108
4.6. Stacks	112
4.7. Exercises	115
Chapter 5. Algebraic spaces	119
5.1. Properties of sheaves and definition of algebraic space	120
5.2. Algebraic spaces as sheaf quotients	124
5.3. Examples of algebraic spaces	127
5.4. Basic properties of algebraic spaces	129
5.5. Algebraic spaces are fppf sheaves	134

5.6. Exercises	135
Chapter 6. Invariants and quotients	137
6.1. Review of some commutative algebra	137
6.2. Quotients by finite flat groupoids	139
6.3. Topological properties of algebraic spaces	145
6.4. Schematic open subspaces of algebraic spaces	148
6.5. Exercises	149
Chapter 7. Quasi-coherent sheaves on algebraic spaces	151
7.1. The category of quasi-coherent sheaves	151
7.2. Affine morphisms and Stein factorization	155
7.3. Nilpotent thickenings of schemes	162
7.4. Chow's lemma for algebraic spaces	163
7.5. Finiteness of cohomology	164
7.6. Exercises	167
Chapter 8. Algebraic stacks: Definitions and basic properties	169
8.1. Definition of algebraic stack and fiber products	169
8.2. Properties of algebraic stacks and morphisms between them	175
8.3. Deligne-Mumford stacks	178
8.4. Examples	183
8.5. Exercises	188
Chapter 9. Quasi-coherent sheaves on algebraic stacks	191
9.1. The lisse-étale site	191
9.2. Comparison with simplicial sheaves and the étale topoi	197
9.3. Pulling back quasi-coherent sheaves	203
9.4. Exercises	205
Chapter 10. Basic geometric properties and constructions for stacks	209
10.1. Proper morphisms	209
10.2. Relative Spec and Proj	210
10.3. Root stacks	215
10.4. Exercises	218
Chapter 11. Coarse moduli spaces	221
11.1. Basics on coarse moduli spaces	221
11.2. Proof of the main theorem	222
11.3. Applications of the local structure of coarse moduli spaces	230
11.4. Chow's lemma for Deligne-Mumford stacks and applications	233
11.5. The valuative criterion for properness	235
11.6. Finiteness of cohomology	237
11.7. Exercises	239
Chapter 12. Gerbes	243
12.1. Torsors and H^1	243
12.2. Generalities on gerbes	246
12.3. Gerbes and twisted sheaves	250
12.4. Exercises	254

Chapter 13. Moduli of curves	259
13.1. Moduli of elliptic curves	259
13.2. The stack $\overline{\mathcal{M}}_g$.	266
13.3. Moduli of stable maps	278
13.4. Exercises	282
Appendix A. Glossary of category theory	285
Bibliography	291
Index of Notation	295
Index of Terminology	297