

Index

- (ρ_-, ρ_+) -embedding, 258
- (ρ_-, ρ_+) -transformation, 258
- $A(w)$, algebraic area of a word, 239
- $CAT(\kappa)$ triangle, 83
- $CAT(\kappa)$ -domain, 83
- $CAT(\kappa)$ -space, 83
- G -cell complex, 147
- G -invariant kernel, 629
- $Hom_G(C_*, A)$, 162
- $Hom_G(M, N)$, 161
- L -Lipschitz cellular map, 302
- L -action, 620
- R -centroid of a triangle, 385
- R -neighborhood of a subcomplex, 302
- $S_{m,k}$, free solvable group, 491
- Ar_δ , the δ -filling area, 319
- \mathbb{Q}_p , the field of p -adic numbers, 36
- $Trip(Z)$, 411
- α -universal tree, 368
- δ -disk, 318
- δ -filling disk, 318
- δ -filling radius function r_δ , 319
- δ -isoperimetric function Ar_δ , 319
- δ -loop, 318
- κ -compatible triangle, 82
- λ -constriction function, 430
- μ -simply connected space, 319
- ω -limit, 337
- σ -algebra, 3
- σ -compact, 7
- ε -separated subset, 29
- i -th skeleton of a simplicial complex, 18
- k -step nilpotent group, 474
- m -length, 507
- m -separating subset, 540
- n -fold left commutator $[x_1, \dots, x_n]$, 474
- r -dense subset, 29, 254
- x^y , 119

- a-T-menable, 625
- abelian group, 124
- abelianization, 125
- absolutely continuous function, 703
- abstract L^p -space, 619

- abstract commensurator, 124
- accessible group, 666
- ACL property for quasiconformal maps, 706
- action without inversions, 147
- adjoint representation, 136
- affine algebraic set, 141
- affine variety, 141
- Alexandrov's characterization of covering dimension, 35
- algebra of subsets, 3
- algebraic set, 141
- algebraic subgroup, 145
- almost finitely presented group (afp), 664
- almost invariant vector, 627
- almost regular cell complex, 20
- amalgamated free product, 213
- amenable graph, 573
- amenable group, 584
- angle deficit, 72
- approximate equivalence \approx of functions, 2
- approximate isomorphism, 314
- approximately continuous function, 702
- approximation of triangles by tripods, 384
- archimedean norm, 36
- area of hyperbolic triangle, 107
- arithmetic subgroup, 452
- Artin groups, 210
- ascending chain condition, 141
- Assouad's L^p -embedding theorem, 361
- asymptotic characterization of hyperbolic spaces, 423
- asymptotic cone, 352
- asymptotic equality \asymp , 2
- asymptotic inequality \preceq , 2
- asymptotic rays, 88
- asymptotic rays in a hyperbolic space, 390
- augmentation, 160
- Axiom of Choice, 333

- Baire space, 131
- Banach lattice, 617
- Banach sublattice, 617
- Banach–Tarski paradox, 567
- bar notation, 163

- base of a filter, 335
- based ultralimit, 345
- basic commutator identities, 474
- basis of a free abelian group, 465
- Bass–Guivarc’h Theorem, 514
- Bass–Serre correspondence, 221
- Baumslag–Solitar group, 211
- Beltrami differential, 725
- Beltrami equation, 725
- bi-Lipschitz map, 33
- Bianchi group, 453
- bimodule, 161
- bipartite graph, 17
- Bishop–Gromov–Günther inequality, 72
- Borel construction, 149
- Borel subset, 3
- Borel’s theorem on actions of solvable groups, 531
- boundary extension of a quasiaction, 406
- boundary operator, 161
- bounded cohomology groups, 165
- bounded generation property, 203
- bounded geometry cell complex, 76
- bounded kernel, 629
- bounded measurable conformal structure, 722
- Bourdon–Pajot strong QI rigidity theorem for hyperbolic buildings, 758
- Bowditch convergence group theorem, 412
- branch-point of a real tree, 368
- brick, 319
- Burnside group, 212
- Busemann function, 89
- Buser’s inequality, 72

- canonical collapsing map, 648
- canonical resolution of a G -tree, 648
- Cartan decomposition, 45
- Cartan’s fixed-point theorem, 87
- Cartan–Hadamard theorem, 85
- Cayley graph, 228
- cell complex, 19
- cellular action, 147
- center, 120
- centroid of a triangle, 396
- Chabauty topology, 29
- characteristic subgroup, 120
- Cheeger constant $h(M)$, 70
- chordal metric on projective space, 45
- class stable with respect to ultralimits, 360
- classical topology, 146
- classifying space, 148
- closed lcs-generating set, 507
- co-large group, 606
- coarea formula, 67
- coarse Arzela–Ascoli theorem, 261
- coarse Besikovitch inequality, 331
- coarse convergence, 260
- coarse homotopy-equivalence, 299
- coarse isoperimetric function Ar_δ , 319
- coarse Lipschitz map, 252
- coarsely k -connected space, 298
- coarsely homotopic maps, 299
- coarsely separating subcomplex, 315
- coarsely simply connected space, 298
- cobounded action, 132
- cobounded quasiaction, 272
- cocompact action, 132
- cocycle of a transversal graph, 653
- cohomology with compact support, 18
- collapsing map for geodesic triangles, 383
- combinatorial volume, 235
- commutator, 119
- commutator norm, 125
- commutator subgroup, 124
- comparison point, 83
- comparison triangle, 83
- complete normed, 36
- complexity of a transversal graph, 658
- conditionally negative semidefinite kernel, 51
- cone topology, 397
- conformal metrics, 62
- conformally Euclidean metric, 62
- congruent subsets, 565
- conical limit point, 407
- conjugacy problem, 249
- constant ultralimit, 344
- continuous action, 131
- contractibility functions, 303
- convergence group, 411
- convergence property for quasiconformal maps, 711
- convex subset, 26
- convexity of the distance function, 85
- coordinate ring, 141
- covering dimension, 10
- Coxeter group, 210
- critical value of a PC map, 649
- cube complex, 85
- cut-vertex, 658
- CW complex, 19
- cyclic series, 484
- cyclically reduced word, 203

- Dehn function, 239
- derivative of a measure, 702
- derived length, 489
- derived series, 489
- derived subgroup, 124
- diameter of a subcomplex, 302
- differentiability a.e. of quasiconformal maps, 705
- dihedral group, 125
- direct limit, 11
- direct product of groups, 126

- direct sum of groups, 129
- direct system, 11
- directed set, 11
- discrete norm, 36
- disjoint elements of a Banach lattice, 618
- displacement function Δ , 557
- distance function, 23
- distance function in \mathbb{H}^n , 108
- distortion function of a map, 259
- distortion function of a uniformly proper map, 302
- distortion of a subgroup, 284
- divergence of geodesics, 385
- Douady track, 652
- doubling metric space, 552
- dual hyperbolic cosine law, 104
- Dunwoody's accessibility theorem, xix, 667
- Dymarz QI rigidity theorem, 767

- edge-boundary, 16
- edge-group, 214
- Efremovich's quasisymmetry theorem, 712
- embedding problem, 249
- ends of a space, 290
- epimorphism, 119
- equivariant map, 130
- Eskin's strong QI rigidity theorem for non-uniform higher rank lattices, 755
- Eskin–Fisher–Whyte QI rigidity theorem, 766
- essential track, 655
- Euclidean isoperimetric inequality, 70, 434
- exact sequence, 127
- exchange argument, 661
- expanding endomorphism, 278
- exponential growth, 277
- extended Morse lemma, 402
- extension theorem, 404

- Farb–Mosher abelian-by-cyclic QI rigidity theorem, 765
- field of non-standard real numbers \mathbb{R}^ω , 341
- filling order, 323
- filling radius characterization of hyperbolicity, 434
- filling radius characterization of hyperbolicity — strong version, 437
- filling radius function, 320
- filter, 334
- final topology, 12
- finitely additive probability (f.a.p.) measure, 3
- finitely generated group, 199
- finitely presented group, 206
- first variation formula, 104
- fixed-point property *FC*, 623
- footpoint of a horoball, 89
- Fox calculus, 167
- Fox derivative ∂_i , 167

- Fréchet filter, 334
- Fratini subgroup, 202
- free abelian group, 465
- free action, 130
- free group, 204
- free groups QI rigidity theorem, xviii
- free nilpotent group, 478
- free solvable group $S_{m,k}$, 491
- fundamental group of graph of groups, 215
- Følner property, 590
- Følner sequence for a group, 588
- Følner sequence for an action, 588
- Følner sequence for the graph, 573

- gallery in a simplicial complex, 18
- Gauss-Bonnet formula, 71
- generalized Rademacher theorem, 65
- generalized von Dyck group, 210
- generating set, 122
- geodesic, 26
- geodesic metric space, 26
- geodesic triangle, 27
- geometric action, 140
- geometric quasi-action, 272
- geometric realization, 18
- geometric volume of a map, 65
- graph of groups, 214
- Grigorchuk's subexponential growth theorem, xvi
- Gromov boundary, 399
- Gromov product, 368
- Gromov topology, 398
- Gromov's polynomial growth theorem, xvii, 549
- Gromov–Hausdorff distance, 29
- Gromov-hyperbolic space, 369
- Gromov-hyperbolicity is not QI invariant, 371
- group \mathcal{U}_n , 475
- group $\mathbb{E}(G, A)$ of central coextensions, 172
- group action, 129
- group action on a tree, 219
- group coextension, 168
- group cohomology, 162
- group extension, 168
- group of derivations $Der(L, M)$, 166
- group of quasiisometries $QI(X)$, 257
- group of type \mathbf{F}_n , 159
- group ring, 160
- group ring derivation, 167
- growth constant γ_S , 277
- growth function, 275

- Haagerup property, 627
- Hadamard manifold, 85
- Hahn–Banach theorem, 334
- Hall identity, 481
- Hall–Rado marriage theorem, 574
- Hausdorff dimension, 34

- Hausdorff measure, 34
- Hausdorff-convergence, 29
- Heisenberg group, 475
- Hilbert basis theorem, 141
- Hirsch number, 488
- HNN extension, 214
- homogeneous metric space, 25
- Hopf–Rinow theorem, 27
- horoball, 89
- horoball QI extension theorem, 738
- horoballs in \mathbb{H}^n , 110
- horocycle, 89
- horosphere, 89
- horospheres in \mathbb{H}^n , 110
- hyperbolic cosine law, 103
- hyperbolic group, 407
- hyperbolic sine law, 103
- hyperbolic triangle $\triangle ABC$, 105
- hyperbolic trigonometry, 102
- hypersurface, 59

- ideal boundary ∂_∞ of a metric space, 88
- ideal boundary of a hyperbolic group, 410
- ideal point, 88
- ideal triangle, 396
- immersion, 60
- induced intrinsic metric, 26
- infinitely large non-standard real number, 341
- inhomogeneous bar complex, 163
- initial topology, 12
- injectivity radius $\text{InjRad}(p)$, 64
- inner dilatation, 698
- inradius, 315
- inradius of a triangle, 366
- inscribed radius $\text{inrad}(T)$ of a hyperbolic triangle, 116
- integer cohomological dimension, 162
- integer Heisenberg group, 211, 475, 499
- internal subset, 341
- invariant factors, 44
- invariant map, 130
- invariant mean, 584
- invariant measurable conformal structure, 722
- inverse limit, 12
- inverse system, 11
- inversion, 92
- inverted linear map, 745
- irreducible components of a noetherian space, 145
- irreducible decomposition of a noetherian space, 144
- irreducible lattice, 452
- irreducible topological space, 143
- isometric action, 130
- isometric affine action, 624
- isometric embedding, 25
- isometry, 25
- isometry group, 25
- isomorphism of affine varieties, 142
- isomorphism problem, 249
- isoperimetric inequality, 70, 281
- iterated commutator $\llbracket x_1, \dots, x_{2^k} \rrbracket$, 489
- iterated commutator subgroup $G^{(k)}$, 489

- John–Loewner ellipsoid, 720
- Jordan’s theorem, 528
- Jordan–von Neumann characterization of Hilbert spaces, 615

- Kakutani representation theorem, 620
- Kazhdan constant, 627
- Kazhdan pair, 627
- Kazhdan set, 627
- Kazhdan’s Property (T), 627
- kernel, 51
- kernel of a quasiaction, 272
- Kleiner–Leeb central extension theorem, 757
- Kleiner–Leeb strong QI rigidity theorem for buildings, 754
- Kleiner–Leeb strong QI rigidity theorem for symmetric spaces, 754

- lamplighter group, 494
- large group, 606
- lattice in a topological group, 139
- law, 211
- lcs-generating set, 507
- lcs-length, 507
- left Haar measure, 137
- left module, 160
- length metric space, 26
- length of a path, 25
- Lie algebra, 135
- Lie group, 134
- Lie–Ado theorem, 137
- limit geodesic, 346
- limit set of a family of subsets, 353
- linear dilatation, 698
- linear dilatation of a measurable Riemannian metric, 722
- linear isoperimetric inequality, 323
- Lipschitz cellular approximation, 304
- Lipschitz map, 30
- local normed ring, 36
- local-to-global characterization of hyperbolicity, 426
- locally compact space, 7
- locally isomorphic topological groups, 130
- Lorentzian model of \mathbb{H}^n , 98
- Łoś’ theorem, 343
- lower central series, 474
- lower-dimensional coboundaries and cocycles, 163

- lower-dimensional coboundary operators, 163
- Magnus embedding theorem, 493
- maximal dilatation, 698
- maximum principle, 80
- maximum principle for stationary tracks, 660
- mean on a set, 584
- measurable conformal structure, 721
- measurable fundamental set, 137
- measurable Riemann mapping theorem, 726
- measurable Riemannian metric, 63
- measure-equivalent groups, 271
- median algebra, 176
- median point, 175
- median space, 175
- Meeks–Yau trick, 662
- membership problem, 249
- metabelian group, 489
- metric, 23
- metric cell complex, 300
- metric cell complex of bounded geometry, 301
- metric graph, 28
- metric simplicial complex, 74
- metric simplicial complex of bounded geometry, 75
- metric space, 23
- metric space of Alexandrov curvature $\leq \kappa$, 83
- metrically properly action, 132
- Mikhailova’s construction, 284
- Milnor’s construction, 148
- Milnor’s question, 280
- Milnor’s theorem, 517
- Milnor–Schwarz theorem, 262
- Milnor–Wolf theorem on growth of solvable groups, 519
- minimal track, 658
- minsize function, 429
- minsize of a topological triangle, 330
- modified Hausdorff distance, 29
- Moebius transformation, 92
- monomorphism, 119
- Montgomery–Zippin theorem for group actions, 550
- Montgomery–Zippin theorem for locally compact groups, 550
- morphism of affine varieties, 141
- Morse lemma, 373
- Mostow rigidity theorem, xviii, 740
- multiplicity of a cover, 10
- nearest-point projection in a hyperbolic space, 381
- Nielsen–Schreier theorem, 213, 231
- nilpotency class, 474
- noetherian group, 493
- noetherian ring, 141
- noetherian topological space, 142
- non-abelian cohomology, 164
- non-abelian derivation, 164
- non-archimedean norm, 35
- non-standard elements, 341
- non-standard extension, 341
- non-standard induction, 343
- non-standard natural numbers \mathbb{N}^ω , 341
- non-standard product, 343
- norm on a ring, 35
- normally generated subgroup $\langle\langle R \rangle\rangle$, 122
- number of ends, 288
- open map, 61
- order isometric Banach lattices, 618
- order of a group element, 119
- order relation \lesssim for functions, 2
- outer dilatation, 698
- packing number, 552
- Pansu QI rigidity theorem for abelian groups, 561
- Pansu strong QI rigidity theorem for quaternionic-hyperbolic spaces and octonionic hyperbolic plane, 753
- Pansu theorem on asymptotic cones of nilpotent groups, 562
- Papasoglu QI rigidity theorem, 772
- paradoxical action, 566
- paradoxical group, 566
- paradoxical subset, 565
- paradoxical-amenable alternative, 594
- parallelogram identity, 24
- Paulin–Tukia extension theorem, 714
- Peng–Dymarz rigidity theorem, 767
- perfect matching, 574
- perfectly normal, 8
- peripheral horosphere, 461
- piecewise-canonical (PC) map, 649
- piecewise-smooth function, 59
- piecewise-smooth submanifold, 59
- Ping-pong lemma, 223
- Poincaré duality, 314
- Point-selection theorem, 620
- poly- C_∞ group, 484
- polycyclic group, 484
- polynomial distortion in nilpotent groups, 507, 511, 512
- precisely invariant subset, 665
- presentation, 206
- presentation complex, 236
- principal non-abelian derivation, 164
- principal ultrafilter, 336
- product formula, 40
- product metric, 24
- projective space $\mathbb{F}P^n$, 45
- proper action, 131
- proper kernel, 629

- properly discontinuous action, 131
- properly discontinuous quasiaction, 272
- Property FL^p , 639
- Property A of Montgomery and Zippin, 549
- Property FA, 87
- Property FH, 625
- proximal endomorphism, 50
- pull operation for transversal graphs, 656
- pull-back, 62

- QI invariance of cohomological dimension, 312
- QI invariance of Rips-hyperbolicity, 375
- QI invariance of type \mathbf{F}_n , 308
- QI rigid class of groups, 274
- QI rigidity of free groups, 671
- QI rigidity of nilpotent groups, xvii, 561
- quadratic isoperimetric inequality, 323
- quasiaction, 271
- quasiconformal Liouville theorem, 710
- quasiconformal map, 701, 709
- quasiconvex hull, 379
- quasiconvex subgroup, 409
- quasiconvex subset, 379
- quasigeodesic, 253
- quasigeodesic metric space, 254
- quasihomomorphism, 166, 271
- quasiisometric embedding, 252
- quasiisometric metric spaces, 252
- quasiisometric relation, 256
- quasimöbius map, 700
- quasimorphism, 166
- quasisymmetric map, 700
- quaternionic-hyperbolic space, 626

- Rademacher theorem, 31
- Rademacher–Stepanov theorem, 703
- radius of convexity $ConRad(p)$, 64
- random group, 443
- rank of a finitely generated group, 199
- rank of a free abelian group, 465
- real tree, 84
- reduced word, 203
- refinement, 120
- refinement of graphs of groups, 668
- regular cell complex, 20
- regular cellular map, 22, 235
- regular growth theorem, 553
- regular value of a PC map, 649
- relators, 206
- residually finite group, 244
- restriction of scalars, 523
- Riemannian distance function, 63
- Riemannian geodesic, 64
- Riemannian growth function, 68
- Riemannian isometry, 63
- Riemannian length, 63
- Riemannian manifold, 62
- Riemannian manifold of bounded geometry, 73
- Riemannian metric, 61
- right module, 161
- Right-angled Artin group (RAAG), 210
- ring O_p of p -adic integers, 36
- ring derivation, 166
- ring of adeles, 40
- Rips complex, 30
- Rips complex $Rips_R(X)$, 30
- Rips-hyperbolic space, 363
- Rosenblatt theorem on growth of solvable groups, 519
- round-off argument, 662

- Sapir’s asymptotic characterization of virtually nilpotent groups, 562
- Sard’s theorem, 61
- Schwartz QI rigidity theorem, xviii, 733
- Schwartz strong QI rigidity theorem for non-uniform rank 1 lattices, 754
- sectional curvature, 71
- semidirect product, 126
- semisimple Lie group, 136
- separated net, 30
- shallow component, 315
- Shalom–Tao effective polynomial growth theorem, xvii
- shearing automorphism, 170
- Shepherd group, 210
- similarity, 25
- simple Lie group, 136
- simplicial action, 147
- simplicial graph, 13
- simplicial tree, 15
- simplicial volume, 235
- simultaneous conjugacy problem, 249
- Singular Value Decomposition, 44, 698
- small group, 606
- Smith Normal Form, 44
- smooth embedding, 60
- smooth submanifold, 59
- solid hyperbolic triangle $\blacktriangle(A, B, C)$, 105
- solvable group, 489
- space of directions, 368
- space with measured walls, 187
- split exact sequence, 127
- Stallings “Ends of groups” theorem, xix, 296
- Stallings theorem for afp groups, 664
- standard metric on a graph, 27
- standard metric on a simplicial complex, 75
- stationary transversal graph, 659
- Stone–Čech compactification βX , 339
- Strong convergence property, 711
- strongly QI rigid group, 274
- strongly QI rigid metric space, 274
- structure theorem for virtually solvable matrix groups, 533

- subexponential growth, 277
- submedian space, 176
- submersion, 60
- subnormal descending series, 120
- subquadratic isoperimetric inequality
 - characterization of hyperbolicity, 433
- subquadratic isoperimetric inequality
 - characterization of hyperbolicity —
 - strong version, 433
- Sullivan–Tukia theorem, 720
- supramenable group, 596
- surface group, 209
- Sznirelman’s inequality, 34

- Tarski number, 601
- Tarski’s alternative, 594
- thick-thin decomposition, 460
- thin polygon, 364
- thin triangle, 363
- thin triangles property for \mathbb{H}^n , 117
- thinness radius of a triangle, 363
- Tits Alternative, xvi, 537, 546
- topological coupling, 269
- topological group, 130
- topology on the ideal boundary, 88
- topology on the set of ends, 290
- torsion subgroup, 121
- torsion-free group, 121
- totally geodesic, 64
- transversal graph, 652
- tripod, 382
- trivial refinement, 668
- triviality problem, 249
- truncated hyperbolic space, 461
- Tukia’s extension theorem, 719
- Tukia’s QI rigidity theorem, xix, 717
- twisted conjugacy class, 165

- ultrafilter, 335, 336
- ultrafilter lemma, 336
- ultralimit, 337
- ultralimit of a sequence of maps, 346
- ultralimit of a sequence of metric spaces, 344
- ultrametric inequality, 24, 35
- ultrapower, 341
- ultrapower of a group, 343
- ultraproduct, 340
- undistorted subgroup, 284
- uniform convergence action, 411
- uniform Følner property, 606
- uniform lattice, 139
- uniformizer, 36
- uniformly k -connected metric cell complex, 303
- uniformly contractible metric cell complex, 303
- uniformly discrete metric space, 25
- uniformly proper cellular map, 302
- uniformly proper map, 259
- uniformly QI rigid metric space, 274
- unimodular group, 137
- uniqueness of universal trees, 368
- unit ball model of \mathbb{H}^n , 96
- unit speed geodesic, 64
- unoriented graph, 13
- upper central series, 479
- upper half-space model of \mathbb{H}^n , 94

- valence of a real tree, 368
- valency, 14
- valuation, 36
- valuation ring, 36
- Van den Dries–Wilkie theorem on groups of
 - weakly polynomial growth, 549
- van Kampen diagram, 241
- vertex group, 214
- vertex-boundary, 16
- very proximal automorphism, 50
- virtual property, 121
- virtually isomorphic groups, 123
- volume element dV , 64

- wall, 186
- wall pseudo-metric, 187
- weakly paradoxical action, 566
- weakly polynomial growth, 549
- weakly quasimetric map, 699
- Wolf’s theorem, 516
- word, 203
- word metric, 228
- word norm, 228
- word problem, 249
- wreath product, 129

- Xie’s QI rigidity theorem for hyperbolic
 - buildings, 758

- Zariski dense, 142
- Zariski topology, 142
- Zassenhaus neighborhood, 525
- Zassenhaus theorem, 527
- zooming argument, 728, 740, 743