Preface

Graphs are usually represented as geometric objects drawn in the plane, consisting of nodes and curves connecting them. The main message of this book is that *such a representation is not merely a way to visualize the graph, but an important mathematical tool.* It is obvious that this geometry is crucial in engineering if you want to understand rigidity of frameworks and mobility of mechanisms. But even if there is no geometry directly connected to the graph-theoretic problem, a well-chosen geometric embedding has mathematical meaning and applications in proofs and algorithms. This thought emerged in the 1970s, and I found it quite fruitful: Among its first applications, it led to a classification result in the theory of node-coverings, to the theory of orthogonal representations, and through this, to several combinatorial applications of semidefinite optimization.

I have given quite a few courses and lectures about the fast expanding theory of geometric representations of graphs. In 2014, I gave an advanced course on this topic at the Eidgenossische Technische Hochschule (ETH) in Zurich, and the interest and active participation of the audience, faculty, postdocs, and graduate students provided great inspiration. I am most grateful for the hospitality of ETH.

By that time, I had decided that it was worthwhile to merge my handouts and lecture notes about this general topic into a book. This took quite some time; partly because other duties limited the time I could spend on this project, but also because I kept recognizing common concepts that extended through several seemingly unrelated topics, and holding, I hope, the material better together (for example, randomization, non-degeneracy and duality). Many new results and new applications of the topic have also been emerging, even outside mathematics, like in statistical and quantum physics and computer science (learning theory). At some point I had to decide to round things up and publish this book.

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