

Contents

| | |
|---|-----|
| Background and overview | 1 |
| Chapter 0. Introduction | 3 |
| Chapter 1. The major theorems and some background | 7 |
| 1.1. Theorems 1 through 8 | 7 |
| 1.2. Background | 10 |
| 1.3. An outline of the proof | 13 |
| Basics and examples | 19 |
| Chapter 2. Some basic results | 21 |
| 2.1. Preliminary lemmas | 21 |
| 2.2. Solvable components | 27 |
| 2.3. Intrinsic $SL_2[m]$ -components | 34 |
| 2.4. A sufficient condition for quaternion fusion packets | 39 |
| 2.5. Basic results on fusion packets | 39 |
| 2.6. The case $z \in Z(\mathcal{F})$. | 41 |
| 2.7. $\mathcal{F} = S\mathcal{O}_\tau$ | 45 |
| 2.8. Modules for groups with a strongly embedded subgroup | 48 |
| Chapter 3. Results on τ | 53 |
| 3.1. $\Delta(\tau)$, $\eta(\tau)$, and $\mu(\tau)$ | 54 |
| 3.2. The graph \mathcal{A} | 64 |
| 3.3. More basic lemmas | 66 |
| 3.4. Generating \mathcal{F} | 72 |
| Chapter 4. $W(\tau)$ and $M(\tau)$ | 79 |
| 4.1. 3-transposition groups | 79 |
| 4.2. The groups in $M(\tau)$ | 86 |
| 4.3. The groups $\bar{\omega}(\Phi, m)$ | 90 |
| Chapter 5. Some examples | 95 |
| 5.1. AE_n | 96 |
| 5.2. The 2-share of the order of some groups | 102 |
| 5.3. Orthogonal groups and packets | 105 |
| 5.4. Linear, unitary, and symplectic groups and packets | 109 |
| 5.5. Exceptional groups and packets | 113 |
| 5.6. $\mathcal{F}_S(G)$ is simple | 117 |
| 5.7. $L_d^\pi[m]$ and $\bar{\omega}(A_{d-1}, m)$ | 120 |
| 5.8. $\bar{\omega}(D_n, m)$ | 123 |

| | | |
|-----------------------------|---|------------|
| 5.9. | $\bar{\omega}(C_n, m)$ and $2\bar{\omega}(C_n, m)$ | 127 |
| 5.10. | Some constrained examples | 128 |
| 5.11. | Summary of basics | 132 |
| Theorems 2 through 5 | | 135 |
| Chapter 6. | Theorems 2 and 4 | 137 |
| 6.1. | $\mathcal{D}(\tau)^c$ | 137 |
| 6.2. | Beginning the case $z \in O_2(\mathcal{F})$ | 140 |
| 6.3. | The case $E \not\leq N_G(K)$ | 146 |
| 6.4. | Subnormal closure | 159 |
| 6.5. | $F^*(\mathcal{F})$ | 162 |
| 6.6. | z not in $O_2(\mathcal{F})$ | 166 |
| 6.7. | The proof of Theorem 2 | 170 |
| Chapter 7. | Theorems 3 and 5 | 173 |
| 7.1. | Packets of width 1 | 173 |
| 7.2. | $\mathcal{A}(z) \neq \emptyset$ | 184 |
| Coconnectedness | | 193 |
| Chapter 8. | τ° not coconnected | 195 |
| 8.1. | \mathcal{D}^c disconnected | 195 |
| Theorem 6 | | 203 |
| Chapter 9. | $\Omega = \Omega(z)$ of order 2 | 205 |
| 9.1. | $ \Omega(z) = 2$ | 205 |
| 9.2. | Generation when $ \Omega(z) = 2$ | 211 |
| 9.3. | $ \Omega(z) = 2$ and $\mathcal{Z} \cap O(z) \neq \{z\}$ | 215 |
| 9.4. | $ \Omega(z) = 2$ and $\mathcal{D}^*(z) = \mathcal{D}(z)$ | 223 |
| 9.5. | $ \Omega(z) = 2$ and μ isomorphic to S_4 | 240 |
| Chapter 10. | $ \Omega(z) > 2$ | 255 |
| 10.1. | $ \Omega(z) = 4$ and μ isomorphic to $\text{Weyl}(D_4)$ | 255 |
| 10.2. | $ \Omega(z) $ large | 263 |
| Chapter 11. | Some results on generation | 269 |
| 11.1. | $ \Omega(z) = 2$, μ isomorphic to $\text{Weyl}(D_n)$, $n \geq 4$ | 269 |
| 11.2. | Generation | 273 |
| 11.3. | More generation | 289 |
| 11.4. | Essentials and normal subsystems | 292 |
| 11.5. | Generating $\Omega_d^\epsilon[m]$ | 295 |
| 11.6. | Generating AE_k | 300 |
| Chapter 12. | $ \Omega(z) = 2$ and the proof of Theorem 6 | 305 |
| 12.1. | $ \Omega(z) = 2$, μ isomorphic to $\text{Weyl}(D_4)$ | 305 |
| 12.2. | More $ \Omega(z) = 2$ | 312 |
| 12.3. | Completing $ \Omega(z) = 2$ | 325 |
| 12.4. | The proof of Theorem 6 | 338 |

| | |
|---|-----|
| Theorems 7 and 8 | 339 |
| Chapter 13. $ \Omega(z) = 1$ and μ abelian | 341 |
| 13.1. Systems with μ abelian | 341 |
| 13.2. Generic systems with μ abelian | 351 |
| 13.3. Symplectic groups and systems | 360 |
| 13.4. Linear and unitary groups and systems | 362 |
| 13.5. Generating symplectic and linear systems | 366 |
| 13.6. Finishing μ abelian | 371 |
| Chapter 14. More generation | 375 |
| 14.1. A generation lemma | 375 |
| 14.2. A generation lemma for E_8 | 379 |
| Chapter 15. $ \Omega(z) = 1$ and μ nonabelian | 385 |
| 15.1. $ \Omega(z) = 1$ | 385 |
| 15.2. The case $r > 1$ | 389 |
| 15.3. $\Phi = D_n$ | 399 |
| 15.4. $\Phi = D_4$ | 403 |
| 15.5. $\Phi = A_n$ | 410 |
| 15.6. Generating linear systems | 419 |
| 15.7. Wrapping up $\Phi = A_n$ | 422 |
| 15.8. $\Phi = E_n$ | 424 |
| Theorem 1 and the Main Theorem | 431 |
| Chapter 16. Proofs of four theorems | 433 |
| 16.1. The proof of Theorem 1 | 433 |
| 16.2. Proofs of the Main Theorem and Theorems 6, 7, and 8 | 435 |
| 16.3. Lie fusion packets | 436 |
| References and Index | 439 |
| Bibliography | 441 |
| Index | 443 |