Preface

The papers included in this volume are based on talks given at the AMS Special Sections on the Mathematics of Decisions, Elections, and Games at the 2022 Joint Mathematics Meetings. The session was co-organized by Michael A. Jones (Mathematical Reviews), David McCune (William Jewell College), and Jennifer M. Wilson (The New School). Decision theory, voting theory and game theory are three intertwined areas in the mathematical social sciences that involve making optimal decisions in different contexts.

Decision theory studies instances in which individuals make choices, often under uncertainty, while game theory analyzes optimal decisions when outcomes are affected by all the players' actions. Voting theory and its application to elections, considers the outcomes when individual decisions on submitting ballots for candidates are aggregated to determine a social choice. These areas share a characteristic diversity of mathematical techniques encompassing both continuous and discrete analysis, algebraic and graph theoretic approaches, and probabilistic and deterministic reasoning. The papers in this collection span this diversity. We describe the papers below.

When to stop consulting, by Kilgour and Brams, analyzes optimal strategies for a decision maker who must make a binary choice but has the option to consult experts a fixed number of times. Similar to the Secretary Problem, in which an individual must select a job candidate after a sequence of interviews, the Consultation Problem concerns a situation when a sequence of interactions should terminate a process and trigger a decision. The decision maker's decision to either stop or to consult results in a Bayesian decision binary tree in which optimizing the expected utility depends on the cost of the consultation.

Connecting Arrow's theorem, voting theory, and the Traveling Salesperson Problem, by Saari, shows how the analysis of pairwise voting methods and the traveling salesperson problem can be unified and simplified by treating the corresponding weighted graph as a vector space and decomposing it into portions that contain cycles and those that do not. Saari shows that finding a winner from a given profile reduces to analyzing the non-cyclic component of the graph while finding an optimal solution for the traveling salesperson problem can be simplified by analyzing the graph's cyclic component.

Graph theory is also used in *Piercing numbers in circular societies*, by Mazur, Sondjaja, Wright, and Yarnall. In their work, the authors extend the analysis of approval voting to situations when individuals express approval for intervals on a circle. Previous work has analyzed bounds on the number of voters who approve of a single candidate when preferences live on a fixed interval or on a circle. This paper asks a different question: what is the minimum number of candidates needed to satisfy all voters? The authors analyze the situation when individual approval sets are of equal length and consider both deterministic and probabilistic models.

Impossibility theorems involving weakenings of expansion consistency and resoluteness in voting, by Holliday, Norman, Pacuit, and Zahedian, examines how versions of Arrow's axiom of expansion consistency are incompatible with other fundamental axioms in voting theory including anonymity, neutrality and resoluteness. Specifically, the authors consider binary expansion consistency, which states that a candidate who is the social choice from an initial field of candidates, and who beats a new candidate in a pairwise comparison, should remain the social choice when the new candidate is added to the field. Using a SAT solver, the authors show that even weakened versions of resoluteness are incompatible with binary expansion consistency. The proof makes use of a novel method for extending impossibility results on pairwise and majority voting methods to impossibility results on all voting methods, which may have application in future analyses.

Voting on cyclic orders, group theory, and ballots, by Crisman, Holleran, Martin, and Noonan, uses representation theory to analyze both the ballots for voting on cyclic orders and points-based procedures that aggregate such ballots. In this setting, voters provide rankings of the cyclic orders of n candidates. The authors provide a full characterization of points-based voting procedures for two ballot types when $n = 4$ and offer observations for $n = 5$.

Conditions for fairness anomalies in instant-runoff voting, by Graham-Squire, extends previous work that analyzed necessary and sufficient conditions for instant-runoff voting to demonstrate monotonicity paradoxes for three-candidate when ballots are fully-ranked. In this paper, he determines necessary and sufficient conditions for monotonicity paradoxes for three-candidate elections when ballots are not fully ranked, and necessary conditions for elections with four or more candidates. Additionally, he examines no-show anomalies in three-candidates elections for both fully-ranked and not fully-ranked ballots.

In *An iterative procedure for apportionment and its use in the 2016 Georgia Republican primary*, by Jones, McCune and Wilson, the authors analyze an apportionment method used to allocate at-large delegates in Georgia to presidential candidates during the 2016 Republican presidential primary. This method, the Iterated Lower Quota method, is based on a sequence of rounds in which each candidate receive a succession of lower quotas. The authors analyze the properties of this method, including the maximum number of rounds that may occur, and compare it to Hamilton's method. They also discuss its implementation in the 2016 primaries.

Double moves by each player in chess openings make the game fairer, by Brams and Ismail, presents a high-level analysis of work also presented at the 2001 IEEE Conference on Games in which the authors make the case that the apparent advantage in chess given to white (as the first player) can be mitigated by changing the usual sequence of moves, WB/WB/WB/WB, to the sequence WB/BW/WB/WB….

How lies induced cooperation in Golden Balls: A game-theoretic analysis, by Brams and Mor, analyzes a well-known episode from the British TV game show *Golden Balls*, in which one of the two contestants lies about his strategy. This incident transformed a situation which can be modeled by a standard Prisoners'

Dilemma into a game in which the contestants were able to improve their outcomes from the expected one when both players defect.

We would like to thank the authors of these papers, as well as all the presenters in the 2022 AMS Special Section on the Mathematics of Decisions, Elections, and Games, for their interesting contributions and their enthusiastic participation, which was especially appreciated as the 2022 Joint Meetings were moved online. We hope this volume and the session contribute to strengthening the interdisciplinary community of mathematicians and others who work in these areas.