Introduction

Arnaud Beauville's survey *Vector bundles on curves and generalized theta functions*: *recent results and open problems* [**Bea**] appeared 10 years ago. This elegant survey is short (16 pages) but provides a complete introduction to a specific part of algebraic geometry. To emulate his success now, we need many more pages, even though we assume that the reader is already acquainted with the material presented there. Moreover, in Beauville's survey the relation between generalized theta functions and conformal field theories (classical and quantum) was already presented.

Following Beauville's strategy, we do not provide any proofs or motivation. But we would like to present *all constructions* of this large domain of mathematics in such a way that the proofs can be guessed from the geometric picture. Thus this text is not quite a mathematical monograph but rather a digest of a field of mathematical investigations.

In the abelian case (the subject of several beautiful classical books ([**Ba, C, Wi, Fa1**] and many others) by fixing a combinatorial structure (a so-called theta structure of order k), one obtains a special basis in the space of sections of powers of the canonical polarization on Jacobians. These sections can be presented as holomorphic functions on the "abelian Schottky" space $(\mathbb{C}^*)^g$. This fact provides various applications of these concrete analytic formulas to integrable systems, classical mechanics and PDE's (see the references in [**DKN**]).

Our practical goal is to do the same in the non-abelian case, that is, to give the answer to the final question of Beauville's survey (Question 9 in [**Bea**]).

It has been observed many times that the *construction* of theta functions with characteristics is intricately related to the paradigm of the quantization procedure (which is a quantum field theory in dimension 1). New features came from Conformal Field Theory (which is a field theory in dimension $2 = 1 + 1$). This new pathway brings the standard physical paradigm of "symmetries, fields, equations, etc., and gluing properties corresponding to local Lagrangians." New CFT methods provided powerful computational tools while "algebraic geometers would have never dreamed of being able to perform such computations" (A. Beauville).

In the future we hope to extend this digest to a mathematical monograph with the title "VBAC."

Acknowledgments. These notes were written for a series of lectures given at the Centre de recherches mathématiques at the Université de Montréal in September 2002. I am grateful to J. McKay, J. Hurtubise, J. Harnad, D. Korotkin and A. Kokotov who made my stay in Montreal very agreeable and productive. I would like to express my gratitude to my collaborators C. Florentino, J. Mourão, J.P. Nunes and participants of the Quantum Gravity seminar in IST (Lisboa, 2000).

I would like to thank Jean LeTourneux and André Montpetit for their assistance in the preparation of these notes. Special thanks go to my daughter, Yulia Tiourina, for catching numerous mistakes and misprints in this huge text.