

Contents

Introduction	vii
Chapter 1. Unitary Matrix Ensembles	1
1.1. Unitary ensemble with real analytic interaction	1
1.2. Ensemble of eigenvalues	3
1.3. Recurrence equations and discrete string equations for orthogonal polynomials	9
1.4. Deformation equations for the recurrence coefficients	13
1.5. Differential equations and Lax pair for the ψ -functions	16
Chapter 2. The Riemann – Hilbert Problem for Orthogonal Polynomials	19
2.1. The Cauchy transform and its properties	19
2.2. The Riemann – Hilbert problem	20
2.3. Distribution of eigenvalues and equilibrium measure	22
2.4. The Deift – Zhou steepest descent method	27
2.5. Solution of the RHP for $\mathbf{X}_N(z)$	45
2.6. Asymptotics of the recurrence coefficients	47
2.7. Universality in the random matrix model	50
Chapter 3. Discrete Orthogonal Polynomials on an Infinite Lattice	55
3.1. The discrete log gas ensemble	55
3.2. Interpolation problem	56
3.3. Equilibrium measure	57
3.4. The g -function	61
3.5. Reduction of IP to RHP	62
3.6. First transformation of the RHP	65
3.7. Second transformation of the RHP	66
3.8. Model RHP	67
3.9. Parametrix at band-void edge points	68
3.10. Parametrix at the band-saturated region end points	70
3.11. The third and final transformation of the RHP	74
3.12. Asymptotics of recurrence coefficients	75
3.13. Universality in the discrete log gas ensemble	76
Chapter 4. Introduction to the Six-Vertex Model	81
4.1. Definition of the model	81
4.2. Height function and reduction of parameters	82
4.3. Mappings of the six-vertex model onto other ensembles	84
4.4. Exact solution of the six-vertex model for a finite n	88

Chapter 5. The Izergin–Korepin Formula	93
5.1. The Yang–Baxter equation	96
5.2. A proof of Proposition 5.1.1	100
5.3. The recursion equation for Z_n	101
5.4. The inhomogeneous model on the free fermion line	103
5.5. The homogeneous limit	105
Chapter 6. Disordered Phase	109
6.1. Main results	109
6.2. Rescaling of the weight	113
6.3. Equilibrium measure	114
6.4. Riemann–Hilbert analysis	128
6.5. Estimates on the jumps for \mathbf{X}_n	133
6.6. Evaluation of \mathbf{X}_1	135
6.7. Proof of Proposition 6.1.2	139
6.8. The constant term	139
Chapter 7. Antiferroelectric Phase	143
7.1. Introduction	143
7.2. Jacobi theta functions: Definitions and properties	144
7.3. Main result: Asymptotics of the partition function	148
7.4. Equilibrium measure	149
7.5. Riemann–Hilbert analysis	160
7.6. Evaluation of \mathbf{X}_1	173
7.7. The constant term	192
Chapter 8. Ferroelectric Phase	197
8.1. Introduction and formulation of the main results	197
8.2. Meixner polynomials	198
8.3. Two interpolation problems	200
8.4. Evaluation of the ratio h_k/h_k^Q	201
8.5. Evaluation of the constant factor	204
8.6. Ground state configuration	206
Chapter 9. Between the Phases	209
9.1. The critical line between the ferroelectric and disordered phases	209
9.2. The critical line between the antiferroelectric and disordered phases	212
9.3. The order of the phase transitions	214
Bibliography	221