Introduction

1. Mathematics. This book comprises about a dozen of my non-technical papers and a slim book, all written and published during the last three decades or so. It complements the Selecta of my technical papers published in 1996;[1] cf. also the complete list of my publications at the end of this volume.

The present selection should be seen against the background formed by fifty years of research and reflections about our trade as a specimen and a metaphor of the human condition. This attitude is not as esoteric as it might sound, if only because through mathematics, as through poetry and philosophy, dozens of generations communicate to each other, often over the heads of their contemporaries, because in each generation the community of mathematicians is small and scattered all over the world.

Most mathematical research in which I have been engaged started with Algebraic Geometry. Its centerpiece is the study of solutions to systems of polynomial equations with many indeterminates. When equations are given and fixed, we imagine the set of all their solutions, consisting of n-tuples of complex numbers, as a geometric entity, a shape situated in an n-dimensional space, stretching to infinity in some directions and capriciously closing up on itself in others. The variety and complexity of such forms is incomparably richer than anything one could glimpse at modern exhibitions of abstract art, but algebraic geometers managed to find patterns, discover connections and determine the laws of this immense world.

I was primarily interested in applications of Algebraic Geometry to Number Theory and to Theoretical Physics.

One of the oldest problems in number theory, going back to the ancient Greeks and bearing the name of Diophantus of Alexandria (ca. 300 AD), also concerns solutions to polynomial equations, but this time one assumes that the coefficients of the polynomials in question are integers, and one asks:

Is there a solution all of whose coordinates are integers (lattice points) or rational numbers? How many such solutions can one expect to find?

What most fascinated me was the slowly emerging understanding that the answers to these Diophantine questions depend critically on the geometric shape of the space of all complex solutions.

For example, the space of all complex solutions can be two-dimensional, and similar to a sphere, or a torus, or a more complicated surface with two or more handles. The number of the handles is the so-called *genus*, a very robust topological

[1] Y. Manin. *Selected Papers.* World Scientific Series in 20th Century Mathematics, vol. 3, World Sci., Singapore, 1996, xii+600 pp.

invariant of the system of equations, ostensibly having little to do with arithmetic subtleties and discrete lattice points of a Diophantine system.

Nevertheless, the genus essentially determines when the set of rational solutions can be infinite: *never, if there are at least two handles.*

This is the celebrated Mordell Conjecture which I tried to attack in the sixties, with only a partial success. Later I initiated a program which purports to establish, in arbitrary dimension, how some topological characteristics of the variety of complex solutions determine the asymptotic behaviour of the number of those lattice points on this variety that are situated in a cube of growing size.

The mathematical toolkit of Theoretical Physics did not include more than rudimentary Algebraic Geometry until the second half of the twentieth century, when the machinery of quantum fields and especially of quantum strings put these tools into the foreground.

The visual image of the world line of a point-like elementary particle was replaced by that of the world sheet of a small string. Such a world sheet again looks like a (Riemann) surface, and its genus, the number of handles, corresponds to the number of loops in various Feynman amplitudes which, since the 1940's, formed the main theoretical and computational machinery of quantum field theory.

One of my results in this study of quantum strings, which I greatly enjoyed, was the calculation of the so-called Polyakov measure on the moduli spaces of Riemann surfaces. It turned out that this measure can be built up from the same arithmetical components that played a prominent role in Gerd Faltings' final proof of the Mordell Conjecture on Diophantine equations.

In fact this Platonic feeling, that even most abstruse mathematical ideas are somehow predestined to be in harmony with the physical world, always constituted for me one of the most irresistible attractions of our trade.

Stéphane Mallarmé wanted to make us aware that poetry is made of words rather than ideas. To a certain degree, this is true about mathematics as well, but in a more profound sense, it is fundamentally wrong. (I suspect that it is wrong for poetry as well).

As I have recently written in *Mathematical Knowledge*, "mathematicians developed a peculiar discursive behaviour which might be called 'the culture of definitions'. In this culture, many efforts are invested into clarification of the content (semantics) of basic abstract *notions* and syntax of their interrelations, whereas the choice of *words* (and, even to a larger degree, *notations*) for these notions is a secondary matter and largely arbitrary convention, dictated by convenience, aesthetic considerations, and by the desire to invoke appropriate connotations."

Trying to teach myself the linguistic dimension of mathematics, I wrote, primarily for self-education, *A Course in Mathematical Logic*, which appeared in the Springer Graduate Texts series in 1977.

A Russian version of this book was published by "Soviet Radio" in two parts: *Provable and Unprovable* (1979) and *Computable and Uncomputable* (1980). For the second part I wrote an Introduction (reproduced in this volume), at the end of which I briefly discussed the idea of quantum computers as a potentially powerful computational tool. Several years ago I was pleased to learn that my remarks

prompted at least one young researcher in Russia to dedicate himself to this promising field. R. Feynman's paper, published in 1982 in English, which developed similar arguments in considerable detail, became much more influential.[2]

2. Trickster, Language, Consciousness. In fact, language, its origins and functioning, both in contemporary mathematics and mathematical logic, and at its very early pre-historical stages about which we can only speculate, permanently fascinated me.

Moscow of the 1960's–1980's, where I spent the most creative time of my life, was also boiling with humanities studies and quests for the meaning of human culture, history, and psychology. I happily participated in seminars and conferences organized by my friends, linguists and philologists, delivered talks and published amateurish papers on subjects ranging from the trickster figure in mythology to the epistemology of Lévi-Strauss.

For many years I led a seminar in Moscow on psycholinguistics and evolution of mind and consciousness. Among its participants and contributors were linguists, ethnologists, neurobiologists, psychologists and psychiatrists. All of us had different backgrounds and different interests, and we tried to find common viewpoints and problems that might possibly be clarified by throwing together our diverging experiences.

My own inquiry gradually focussed on a project which probably only a dilettante could have conceived. I started imagining in some detail the emergence of language as a system of social behaviour.

I was trying to see through the mist of centuries, far beyond the boundary where the methods of comparative linguistics start failing because of the exponential decay of available data. (For example, nostratic reconstructions refer to a very late epoch of about $(10–13)\cdot 10^3$ yrs BC.) This justified a change from a purely linguistic viewpoint to a psycholinguistic one.

For brevity, I will present some of my musings in the form of a list of rather dry and simplified theses.

(i) In modern societies, there are a (very) few persons whose level of linguistic competence is considerably higher than that not only of laymen but also of practically all other members of society. I have in mind such crystallizers of national languages as Dante, Shakespeare and Pushkin.

I postulated that this was true also at the very early stages of the development of speech. Put somewhat paradoxically, there were people through which a yet unborn language spoke, and this unsystemic speech generated by a mutated brain burst into a non-speaking environment through proto-shamans and proto-poets.

Thus, in the Saussurean idiom, *parole* antedated *langage.*

[2] "Ecce Romanos quoque invasit inane studium supervacua discendi. His diebus audivi quendam referentem quae primus quisque ex Romanus ducibus fecisset: primus navali proelio Duilius vicit, primus Curius Dentatus in triumpho duxit elephantos. [...] non est profutura talis scientia est tamen quae nos speciosa rerum vanitate detineat" (Seneca, *De brevitate vitae*, 13.3.) "Here's how even Romans have been pervaded by a vacuous habit of learning useless things: just these days I overheard someone mentioning who first among the Roman generals did what: Duilius was the first to win a naval battle, Curius Dentatus was the first to lead elephants in a triumphant parade. [...] while it is not a useful knowledge, at least it keeps us amused by the vanity of all things."

(ii) The primary function of developing conscience was not cognitive. It consisted in the introduction of a psychic mechanism that could *temporarily stop inborn behaviourial patterns.*

The primary function of developing speech was to provide a signal system for stopping such instinctive actions; it could be interiorized and thus form a basis of an individual psyche.

Closely connected with this, developing speech provided to the specially endowed individuals a means to control behaviour of other individuals and a means to create the "alternative realities" which developed later into religion, literature, philosophy and science.

(iii) The developing *left brain/right brain* asymmetry accompanying the growth of linguistic competence of Early Man, and probably expressed initially only in a scattered minority of individuals, could easily lead to what in modern terms would be described as a severe neurotic disturbance. (Similar speculations were based on a different material, e.g., changes in sexual behaviour from that characteristic of the animal condition to that of the first human societies.)

At a certain stage of reconstruction, I realized that what I had been imagining was a figure strikingly resembling the mythological trickster. I started studying the literature on tricksters and found, to my delight, that tricksters all over the world seemed to be endowed with special linguistic abilities and were at the same time thoroughly neurotic. I have succinctly described some results of this study in a paper published in 1987 in Russian, in the *Priroda* (Nature) journal of the Russian Academy of Sciences.

Evolution favored the trickster's genes because his prodigious sexual activity was assisted by his manipulative skills. Moreover, the trickster's role of a wise man near the source of power may have given him an additional reproductive advantage.

Only recently have I discovered that approximately at the same time, in 1988, a group of researchers published the book *Machiavellian Intelligence.*[3]

Its content is briefly summarized in [MI2] as follows: "[...] the evolution of the intellect was primarily driven by selection for manipulative, social expertise within groups where the most challenging problem faced by individuals was dealing with their companions." The term "Machiavellian Intelligence" was coined by the authors (or editors) precisely in order to express this manipulative social expertise, and evidence of its important role was already found in societies of primates.

I was happy to learn that my "Trickster" fits exactly this description.

My whole intellectual life was informed by what I have gradually learned to identify as the great Enlightenment project. Crudely put, it is based upon the belief that human reason has the highest value, and that the expansion of knowledge and education will automatically produce better human beings living a better life.

In the best Enlightenment tradition, scientists started studying archaic consciousness because it existed (everything that exists must be understood), and in order to deliver us from the magnetic power it exercised upon us.

But it turned out that archaic consciousness has become only more powerful in modern times. The last century was a time when reason and un-reason,

[3][MI1] *Machiavellian Intelligence: Social expertise and the evolution of intellect in monkeys, apes and humans.* R. W. Byrne, A. Whiten (eds.), Clarendon Press, Oxford, 1988. [MI2] *Machiavellian Intelligence II. Extensions and evaluations.* Andrew Whiten and Richard W. Byrne (eds.), Cambridge Univ. Press, 1997.

equally strong, were developing and coexisting in a monstrously disharmonic symphony. Along with their more beneficial effects, science, politics and art provided humanity with weapons of mass destruction, highly efficient totalitarian forms of self-organization, and the malicious glorification of all of this.

Still, I trust in the Enlightenment project.

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Mitya first suggested the idea of this Selecta, and when I started seriously working on it, produced English versions of several papers that were published only in Russian.

Xenia's advice, critique, encouragement, and love were indispensable, as always.