FOREWORD[1]

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Some mathematicians are birds, others are frogs. Birds fly high in the air and survey broad vistas of mathematics out to the far horizon. They delight in concepts that unify our thinking and bring together diverse problems from different parts of the landscape. Frogs live in the mud below and see only the flowers that grow nearby. They delight in the details of particular objects, and they solve problems one at a time. Manin is a bird. I happen to be a frog, but I am happy to introduce this book which shows us his bird's-eye view of mathematics.

"Mathematics as Metaphor" is a good slogan for birds. It means that the deepest concepts in mathematics are those which link one world of ideas with another. In the seventeenth century, Descartes linked the disparate worlds of algebra and geometry with his concept of coordinates, and Newton linked the worlds of geometry and dynamics with his concept of fluxions, nowadays called calculus. In the nineteenth century, Boole linked the worlds of logic and algebra with his concept of symbolic logic, and Riemann linked the worlds of geometry and analysis with his concept of Riemann surfaces. Coordinates, fluxions, symbolic logic and Riemann surfaces are all metaphors, extending the meanings of words from familiar to unfamiliar contexts. Manin sees the future of mathematics as an exploration of metaphors that are already visible but not yet understood. The deepest such metaphor is the similarity in structure of ideas between number theory and physics. In both fields he sees tantalizing glimpses of parallel concepts, symmetries linking the continuous with the discrete. He looks forward to a unification which he calls the quantization of mathematics.

Manin disagrees with the widely-accepted story that Hilbert set the agenda for the mathematics of the twentieth century when he presented his famous list of twenty-three unsolved problems to the International Congress of Mathematicians in Paris in 1900. Manin sees the important advances in mathematics coming from programs, not from problems. Problems are usually solved by applying old ideas in new ways. Programs of research are the nurseries where new ideas are born. He sees the Bourbaki program, rewriting the whole of mathematics in a more abstract language, as the source of many of the new ideas of the twentieth century. He sees the Langlands program, unifying number theory with geometry, as a promising source of new ideas for the twenty-first. People who solve famous unsolved problems may win big prizes, but people who start new programs are the real pioneers.

One of the richest sources of ideas for the twentieth century was the program of research started by Georg Cantor in the nineteenth, exploring the world of infinite sets and infinite cardinals and ordinals. Hilbert picked out of this program a particular problem, the proof or disproof of the continuum hypothesis, which became number one on his list. The problem turned out to have an answer that was deeper and more significant than Hilbert had imagined. Kurt Gödel in 1938 showed that the hypothesis could never be disproved, and Paul Cohen in 1963 showed that it could never be proved. The continuum hypothesis became an example of an undecidable mathematical statement, demonstrating that no single set of axioms can encompass the whole of mathematics. Manin is saying that the continuum hypothesis itself turned out to be unimportant. Today few mathematicians care whether the continuum hypothesis is true or false. The important progress was the understanding that many alternative sets of axioms may serve as foundations of mathematics. This understanding grew out of Cantor's program as a whole, not out of the particular problem of the continuum hypothesis.

In his essay discussing Cantor's program, Manin brings together two historically separate domains of mathematics, the theory of infinite numbers created by Cantor in the nineteenth century, and the theory of finite computability created by Alan Turing in the twentieth. Manin brings these two worlds together with a single statement:

"2^x is considerably larger than x".

He remarks that deep mathematics begins as soon as we try to give a precise meaning to that innocent word, "considerably". When x is countably infinite, the statement becomes Cantor's Continuum Hypothesis, the hypothesis that the set of points on a line is the smallest infinite set larger than the set of whole numbers.

When x is large but finite, 2^x is roughly the number of instances that must be checked in order to find a number with a given generic property among all integers having binary expansion of length not exceeding x (or to prove that no such number exists). The statement that 2^x is considerably larger than x means that most questions involving x unknowns, each taking the values one or zero, cannot be answered by a digital computer in a reasonable time (a time polynomially dependent on the length of the input). This statement is the basis of the modern theory of computability. The problem of deciding which arithmetical functions are in principle computable has no general answer. Manin suggests that this is no accident. There is a formal similarity between the Cantor theory of infinite sets and the Turing theory of computability of arithmetic functions. In both theories, undecidability is the rule rather than the exception, and the proofs of undecidability are formally similar. Each of the two theories is a metaphor for the other.

This book contains eleven mathematical essays that give us the core of Manin's thinking, and five non-mathematical essays that give us a glimpse of his intellectual hobbies. The most substantial of the mathematical essays is "Mathematics and Physics", a reprint of a book originally published in Russian in 1979 and in English in 1981. I reviewed it for *Mathematical Intelligencer* in 1983. Here are a few sentences from my review: "Manin's purpose was to make the thought processes of physics intelligible to mathematicians. He achieves this purpose by skillful selection of examples. Incidentally, by his style of writing and thinking, he makes the thought processes of a mathematician intelligible to physicists. He does not try to abolish or blur the distinction between mathematical and physical understanding. One of the many virtues of his book is that it leaves the central mystery, the miraculous effectiveness of mathematics as a tool for the understanding of nature, unexplained and unobscured". The essay is only 54 pages long, but it compresses into this narrow frame a lucid exposition of an astonishingly wide range of topics. To give one example among many, here is Manin's comment on the Feynman integral, a mathematically ill-defined expression which is used habitually by physicists to describe quantum processes: "In the prehistory of integral calculus, an important place is occupied by the remarkable work of Kepler, ‘Stereometry of Wine Barrels’. Integrals that give the volume of solids of revolution used in commerce were calculated in this work at a time when the general definition of an integral had not yet appeared. The mathematical theory of Feynman’s magnificent integrals, which physicists write in vast numbers, is not really far removed from the stereometry of wine barrels”. Manin imagines a future revolution in mathematics, like Newton’s invention of calculus, making Feynman integrals as solid and unambiguous as wine barrels.

Manin is a professional mathematician, and his book is mainly about mathematics. It may come as a surprise to Western readers that he writes with equal eloquence about other subjects such as the collective unconscious, the origin of human language, the psychology of autism, and the role of the trickster in the mythology of many cultures. To his compatriots in Russia, such many-sided interests and expertise would come as no surprise. Russian intellectuals maintain the proud tradition of the old Russian intelligentsia, with scientists and poets and artists and musicians belonging to a single community. They are still today, as we see them in the plays of Chekhov, a group of idealists bound together by their alienation from a superstitious society and a capricious government. In Russia, mathematicians and composers and film-producers talk to one another, walk together in the snow on winter nights, sit together over a bottle of wine, and share each others’ thoughts.

One of Manin’s hobbies is the theory of archetypes invented by the Swiss psychologist Carl Jung. An archetype, according to Jung, is a mental image rooted in a collective unconscious that we all share. The intense emotions that archetypes carry with them are relics of lost memories of collective joy and suffering. Manin is saying that we do not need to accept Jung’s theory as true in order to find it illuminating. The mythological trickster is Manin’s favorite among the archetypes that he describes. At the beginning of Western literature, Achilles and Hector, the heroes of the Iliad, fulfill their tragic destinies and go to their noble deaths. Next comes Odysseus, the hero of the Odyssey, the trickster who survives. Odysseus shows us how to deal with a bad situation by being clever. After ten years of heroic and bloody stalemate, he finally brings the Trojan War to an end by building a wooden horse and stuffing it with well-armed soldiers. The Trojans are tricked into bringing the horse into the city, the soldiers jump out and take the Trojans by surprise, and the city falls. Manin’s “Mythological Trickster” essay shows that the archetype of the trickster is more ancient than Western literature, going back into the myths and legends of pre-literate peoples all over the world. In many of the old legends, the trickster appears as one of a pair of brothers. The older brother is the chief, the dignified and heroic founder of the tribe, the embodiment of virtue and justice. The younger brother is the renegade, the daredevil who plays tricks and breaks the rules, the one who makes fun of the chief and is rarely punished. In the animistic cultures of Siberia before the advent of Christianity, it often happened that a tribe was ruled by two brothers, the chief and the shaman, with the shaman in the role of trickster. Manin also tells the story of Wakdjunkaga, a trickster who appears in legends of the Winnebago Indians. He was adopted by Jung as a type specimen of the trickster archetype. Another example of a chief-and-trickster pair of brothers is to be found in the biblical book of Exodus. Moses leads the children of Israel out of Egypt while his brother Aaron plays tricks to punish and confuse the Egyptian Pharaoh.

Clara Park, professor of English at Williams College, published an essay with the title “No Time for Comedy” in 1979 in the Hudson Review, describing the role of the trickster in literature. Every comic hero is a trickster. Park’s essay says, with eloquence and insight, that our modern literature has too much tragedy and not enough comedy. It happens, not by accident, that Park is a good friend of Manin. Park published in 1967 a book with the title, “The Siege”, a classic account of the upbringing of her autistic daughter. Manin has always had an intense interest in autism as a window into the working of the human mind. An autistic child is in some sense a pure intelligence, seeing the world undistorted by the human emotions and relationships that normal people experience. An autistic child, like Euclid, sees beauty bare. Park reports that her daughter used the word “Heptagon” correctly before she used the word “Yes”. Because of her unawareness of normal human constraints and conventions, an autistic child shares some of the qualities of a trickster. Manin’s essay “It is Still Love” is a review of Park’s book, published in the Russian magazine *Priroda*. He paints a vivid picture of this extraordinary child and her mother, the mother always intensely involved with the child but preserving a Brechtian distance as she observes and describes her slow awakening.

More than thirty years ago, the singer Monique Morelli made a recording of songs with words by Pierre MacOrlan. One of the songs is “La Ville Morte”, the dead city, with a haunting melody tuned to Morelli’s deep contralto, with an accordion singing counterpoint to the voice, and with verbal images of extraordinary intensity. Printed on the page, the words are nothing special:

“En pénétrant dans la ville morte,
Je tenait Margot par le main...
Nous marchions de la nécropole,
Les pieds brisés et sans parole,
Devant ces portes sans cadole,
Devant ces trous indéfinis,
Devant ces portes sans parole
Et ces poubelles pleines de cris”.

“As we entered the dead city, I held Margot by the hand... We walked from the graveyard on our bruised feet, without a word, passing by these doors without locks, these vaguely glimpsed holes, these doors without a word, these garbage-cans full of screams”.

I can never listen to that song without a disproportionate intensity of feeling. I often ask myself why the simple words of the song seem to resonate with some deep level of unconscious memory, as if the souls of the departed are speaking through Morelli’s music. And now unexpectedly in this book I find an answer to my question. In his short essay, “The Empty City Archetype”, Manin describes how the archetype of the dead city appears again and again in the creations of architecture, literature, art and film, from ancient to modern times, ever since human beings began to congregate in cities, ever since other human beings began to congregate in armies to ravage and destroy them. The character who speaks to us in MacOrlan’s song is an old soldier who has long ago been part of an army of occupation. After he has walked with his wife through the dust and ashes of the dead city, he hears once more:

“Chansons de charme d’un clairon
Qui fleurissait une heure lointaine
Dans un rêve de garnison”.

“The magic calls of a bugle that came to life for an hour in an old soldier’s dream”.

The words of MacOrlan and the voice of Morelli seem to be bringing to life a dream from our collective unconscious, a dream of an old soldier wandering through a dead city. The concept of the collective unconscious may be as mythical as the concept of the dead city. Manin’s essay describes the subtle light that these two possibly mythical concepts throw upon each other. He describes the collective unconscious as an irrational force that powerfully pulls us toward death and destruction. The archetype of the dead city is a distillation of the agonies of hundreds of real cities that have been destroyed since cities and marauding armies were invented. Our only way of escape from the insanity of the collective unconscious is a collective consciousness of sanity, based upon hope and reason. The great task that faces our contemporary civilization is to create such a collective consciousness.

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Mathematics as Metaphor

Ordre. [...] *Je sais un peu ce que c'est et combien peu de gens l'entendent. Nulle science humaine ne le peut garder. Saint Thomas ne l'a pas gardé. La mathématique le garde, mais elle est inutile en sa profondeur.*

Pascal, *Pensées*

Introduction

When H. Poincaré first published in 1902 his book *La Science et l'hypothèse*, it became a bestseller. The first chapter of this book was devoted to the nature of mathematical reasoning. Poincaré discussed an old philosophical controversy whether mathematical knowledge could be reduced to long chains of tautological transformations of some basic ("synthetic") truths or it contained something more. He argued that the creative power of mathematics was due to a free choice of the initial hypotheses-definitions which were later on constrained by the comparison of deductions with the observable world.

The society of our days seems to be much less interested in the philosophical subtleties than Poincaré's contemporaries. I do not want to say that science itself became less popular. Such books as S. Weinberg's *The first three minutes* and S. W. Hawking's *A brief history of time* are sold by hundreds of thousands and favorably reviewed in widely distributed newspapers. What has changed is the general mood. The paradoxality of the new physical theories is perceived less dramatically and more pragmatically. (We can note that the perception of visual arts evolved in much the same way: if the first exhibitions of Impressionists were a kind of spiritual revolution, each new wave of the post-war avant-garde immediately acquired family traits of academism.)

In this atmosphere, the heated discussions of the bygone days on the foundational crisis of mathematics and the nature of infinity seem almost irrelevant and certainly inappropriate. The audience responds much livelier to the opinions about school education or a new generation of computers.

This is why I have decided to present at this section an unassuming essay in which our science is considered as a specialized dialect of the natural language, and its functioning as a special case of speech. This implies certain suggestions about high school and University training.

First published in *Proceedings of the International Congress of Mathematicians*, Kyoto, Japan, 1990, Volume II, pp. 1665–1671.

Metaphorism

The word "metaphor" is used here in a non-technical sense, which is best rendered by the following quotations from James P. Carse's book *Finite and infinite Games*:

"Metaphor is the joining of like to unlike such that one can never become the other."

"At its root all language has the character of metaphor, because no matter what it intends to do, it remains language, and remains absolutely unlike whatever it is about".

"The unspeakability of nature is the very possibility of language".

Considering mathematics as a metaphor, I want to stress that the interpretation of mathematical knowledge is a highly creative act. In a way, mathematics is a novel about Nature and Humankind. One cannot tell precisely what mathematics teaches us, in much the same way as one cannot tell what exactly we are taught by *War and Peace*. The teaching itself is submerged in the act of re-thinking this teaching.

This opinion seemingly disagrees with the time-honored tradition of applied mathematics in scientific and technological calculations.

In fact, I want only to restore a certain balance between the technological and the humanitarian sides of mathematics.

Two Examples

Let me try to illustrate the metaphoric potential of mathematics by discussing two disjoint subjects: the Kolmogorov complexity and the "Dictator Theorem" due to K. Arrow.

i) Kolmogorov's complexity of a natural number N is the length of a shortest program P that can generate N, or the length of a shortest code of N. A reader should imagine a way of coding integers which is a partial recursive function $f(P)$ taking natural values. Kolmogorov's theorem states that among all such functions there exist the most economical ones in the following sense: if $C_f(N)$ is the minimal value of P such that $f(P) = N$, then $C_f(N) \leq \text{const}\, C_g(N)$, where const depends only on f, g but not N.

Since P can be reconstructed from its binary notation, the length K of the shortest program generating N is bounded by $\log_2 C_f(N)$. This function, or rather the class of all such functions defined up to a bounded summand, is the Kolmogorov complexity.

First of all, $K(N) \leq \log N + \text{const}$. Of course, this conforms nicely with the historical successes of the place-value notation systems which provided us with the number generating programs of logarithmic length. However, there are arbitrary large integers whose Kolmogorov's complexity is much smaller than the length of their decimal or binary notation, e.g., $K(10^{10^N}) \leq K(N) + \text{const}$. In general, when we use large numbers at all, we seemingly use only those which have a relatively small Kolmogorov's complexity. Even decimals of π which are, probably, the longest explicit numbers ever produced by mathematicians, are Kolmogorov simple because $K([10^N \pi]) \leq \log N + \text{const}$. In general, small Kolmogorov complexity = high degree of organization.

On the other hand, almost all integers N have the complexity close to $\log N$. For example, if $f(P) = N$ for an optimal f, then $K(P)$ is equivalent to $\log P$. Such integers have many remarkable properties which we usually connect with "randomness".

Second, Kolmogorov's complexity can be easily defined for discrete objects which are not numbers, for example, Russian or English texts. Therefore, *War and Peace* has a pretty well-defined measure of its complexity; the indeterminancy is connected with the choice of an optimal coding and seems to be pretty small if one chooses one of the small number of reasonable codings.

From this viewpoint, is *War and Peace* a highly organized or an almost random combinatorial object?

Third, Kolmogorov's complexity is a non-computable function. More precisely, if f is optimal, there is no recursive function $G(N)$ which would differ from $K(N)$ by $O(1)$. One can only bound complexity by computable functions.

I feel that Kolmogorov's complexity is a notion that is very essential to keep in mind in any discussion of the nature of human knowledge.

As long as the content of our knowledge is expressed symbolically (verbally, digitally, ...), there are physical restrictions on the volume of information that can be kept and handled. We always rely upon various methods of information compressing. Kolmogorov complexity puts absolute restrictions on the efficiency of such a compression. When we speak, say, of physical laws, expressed by the equations of motion, we mean that a precise description of the behaviour of a physical system can be obtained by translating these laws into a computer program. But the complexity of laws we can discover and use is clearly bounded. Can we be sure that there are no laws of arbitrary high complexity, even governing the "elementary" systems?

At this point, our discussion becomes totally un-mathematical, and before a mathematically-minded audience I must stop here. But such is the fate of any metaphor.

ii) Arrow's Dictator Theorem was discovered around 1950. Mathematically, it is a combinatorial statement describing certain functions with values in binary relations. Intuitively, it is a formalized discussion of the problem of Social Choice. Suppose that a lawmaker has to establish a law which governs the processing of individual wills of voters into a collective decision. If the voters are asked to choose one of the two alternatives, the standard solution is to do it by the majority of votes. However, usually there are more than two alternatives (imagine the funds allocation problems), and voters may be asked to order them according to their preferences. What should be the algorithm extracting the collective preference from any set of individual preferences? Arrow considered algorithms satisfying some natural and democratic axioms (e.g., when majority prefers A to B, the society prefers A to B). Nevertheless, he discovered that when there are more than two alternatives, the only way to achieve a solution is to nominate a member of the society ("the Dictator") and in uncertain cases to equate his personal preference order to the social one. (Actually, this is one of the versions of Arrow's theorem discovered later. Also, it refers to the case of a finite society; in the infinite case, the social decisions can be made by ultrafilters, appropriately called "the ruling hierarchies".)

In a way, this theorem illustrates the content of Jean-Jacque Rousseau's idea of a Contrat Social.

The fundamental intrinsic inconsistency of the image of the ideal democratic choice can be illustrated by the following story referring to three voters and three alternatives. It is the story of three knights errant at the cross-roads with a stone before them. The inscription on the stone prophesies only losses: who goes to the left will lose his sword; who goes to the right will lose his horse; who goes straight will lose his head. The knights dismount and start taking council. In a Russian version of this story, the knights have names and personalities: the youngest and ardent Alyosha Popovich, the eldest and wisest Dobrynya Nikitich, and the slow peasant Ilya Muromets. So Alyosha values sword more than horse, and horse more than his head; Dobrynya values most his head, then his sword, then his horse; and Ilya prefers his horse to head to sword.

A reader will note that the three individual preference orders constitute one and the same cyclic order on the set of alternatives. As a result, one can decide by majority the choice between any two of the alternatives, but the union of these decisions will be inconsistent: the democratic procedure cannot provide us with a well-ordered list. The knights sigh and delegate the decision-making power to Dobrynya.

Does the Arrow theorem tell us something that we did not know beforehand? Yes, I think it does if we are ready to discuss it seriously, that is, to look closely at the combinatorial proof, to imagine the possible real life content of various assumptions and elementary logical steps made on the way, in general, to enhance our imprecise imagination by the rigid logic of a mathematical reasoning. We can understand better, for example, some tricks of policy-making and some pitfalls the society can leap whole-heartedly into (like accepting without questioning a list of alternatives imposed by a ruling hierarchy, although precisely the compilation of this list can be the central issue of the social decision making).

At this stage, we come to the main topic of our discussion: what distinguishes a mathematical discourse from a natural language discourse, why the Pascalian "ordre" came to reign over our specialized symbolic activities, and is it truly so "useless in its profundity"?

Language and Mathematics

A very interesting chapter of the interaction between mathematics and humanities started about thirty years ago when the first serious attempts of automatic translation were made. These first attempts were a painful failure, at least so for many optimists who believed that in this domain, there are no fundamental obstacles, and it remains only to overcome technical difficulties connected with the sheer amount of information to be processed. In other words, they took for granted that the translation is in principle performed by a not very complex algorithm which only must be made explicit and then implemented as a computer program.

This assumption is a nice example of a mathematical metaphor (actually, a specialization of the general "computer metaphor" used in the brain sciences).

This metaphor proved to be extremely fruitful for the theoretical linguistics in general because it forced linguists to start describing vocabulary, semantics, accidence, and syntax of human languages with an unprecedented degree of explicitness and completeness. Some totally new notions and tools were discovered thanks to this program.

However, the successes of the automatic translation itself were (and still are) scanty. It became clear that written human speech is an extremely bad input data for any algorithmic processing planned as a translation or even as a logical deduction. (I add this proviso because there is nothing special in human speech considered as a material for, say, statistical studies.)

This fact can be considered as a universal property of human languages, and it deserves some attention. One must first of all reject as too naive a usual explanation that the universe of meanings of a human language is too vast and poorly structured to admit a well-organized metalanguage describing this world. The point is that even if we severely restrict this universe to the subset of arithmetic of small integer quantities, we shall still have to face the same difficulty. In fact, this difficulty was a decisive reason for the crystallization of the whole system of arithmetical notation and the basic algorithms of calculation, and later on of symbolic algebra. Even the vocabulary of elementary arithmetic in human language is basically archaic: the finite natural series of primitive societies "one, two, three, indefinitely many" is reproduced in the exponential scale in our "thousand, million, billion, zillion". The expressions for relatively small numbers like "1989" are actually names of the decimal notation and not of the numbers themselves.

The advantage of F. Víete's algebra over the semi-verbal algebra of Diophantus was due not to the fact that it could express new meanings but to the incomparably greater susceptibility to the algorithmic processing ("identical transformations" of our high school algebra).

The rupture of the intuitive and emotional ties between a text and its producer/user so characteristic of the language of science was compensated by the new computational automatisms. In their (albeit restricted) domain they proved to be infinitely more efficient than the traditional Platonian and Aristotelian culture of everyday language discourse. Why then are our scientific papers still written as a disorganized mixture of words and formulas? Partly because we still need those emotional ties; partly because some meanings (like human values) are best rendered in human language. But even as a medium of scientific speech, human language has some inherent advantages: appealing to the spatial and qualitative imagination, it helps to understand "structurally stable" properties like the number of free parameters (dimension), existence of extrema, symmetries. To put it bluntly, it makes possible the metaphorical use of science.

Metaphor and Proof

The views professed here can be considered in relation to the high school and graduate curriculum.

The general mathematical education of the first half of this century was application oriented. It provided the basic minimum for the practical life problems and a smooth transition to the study of engineering and scientific calculations at the college level. The break of this curriculum with the activity of professional mathematicians became more and more pronounced. As is well known, this brought the reaction in the form of NewMath in the USA and similar programs in other countries. These programs introduced into high school mathematics the notions and principles borrowed from professionals: set theory, axiomatic presentation of proofs, strict culture of definitions.

NewMath became widely accepted but its expansion was accompanied by the protesting voices which in the 70s and 80s merged into a loud chorus. The critics disagreed with the basic arguments of the NewMath proponents. Leaving aside the objections based upon the data from cognitive sciences and learning psychology I shall only recall those concerning the general evaluation of the role of the proof in mathematics.

The one pole is represented by the well-known statement due to Nicolas Bourbaki: “Dépuis les Grécs, qui dit Mathématiques, dit demonstration”. According to this perception, the rigorous proof was made a matter of principle in the NewMath programs. It was argued that: a) a proof helps to understand a mathematical fact; b) a rigorous proof is the most essential component of modern professional mathematics; c) mathematics possesses the universally recognized criteria of rigour.

These views were extensively criticised, e.g., by Gila Hanna in the book *Rigorous Proof in Mathematics Education*, OISE Press, Ontario 1983. In particular, Gila Hanna pointed out that mathematicians are far from unanimous in accepting the criteria of rigour (referring to quarrels between logicists, formalists and intuitionists), and that working mathematicians constantly break all rules in the book.

In my opinion, this is irrelevant.

What is relevant is the imbalance between various basic values which is produced by the emphasis on proof. Proof itself is a derivate of the notion of “truth”. There are a lot of values besides truth, among them “activities”, “beauty” and “understanding”, which are essential in high school teaching and later. Neglecting precisely these values, a teacher (or a university professor) tragically fails. Unfortunately, this also is not universally recognized. A sociological analysis of the controversies around the Catastrophe Theory of René Thom shows exactly that the shift of orientation from the formal truth to understanding provoked such a sharp criticism. But of course, the Catastrophe Theory is one of the developed mathematical metaphors and should only be judged as such.

Pedagogically, a proof is just one of the genres of a mathematical text. There are many different genres: a calculation, a commented sketch, a computer program, a description of an algorithmic language, or such a neglected kind as a discussion of the connections between a formal definition and intuitive notions. Every genre has its own laws, in particular, laws of rigour, which only are not codified because they were not paid a special attention.

A central problem of a teacher is to demonstrate at the restricted area of his or her course the variety of types of mathematical activities and underlying value orientations. Of course, this variety is hierarchically organized. The goals may vary from achieving an elementary arithmetical and logical literacy to programming skills, and from the simplest everyday problems to the principles of modern scientific thinking. In the spectrum of these goals, the emphasis on the norms of “rigorous proof” can safely occupy a peripheral position.

But having said all this, I must stress that my argumentation by no means undermines the ideal of a rigorous mathematical reasoning. This ideal is a fundamental constituting principle of mathematics, and in this sense Bourbaki is certainly right. Having no external object of study, being based on a consensus of a restricted circle of devotees, mathematics could not develop without the permanent control of rigid rules of the game. Applicability of mathematics in the strict sense of this word (like its indispensability in the Apollo project) is due to our ability to control a series of symbolic manipulations of fantastic length.

The existence of this ideal is far more essential than its unattainability. The freedom of mathematics (G. Cantor) can only develop in the limits of iron necessity. The hardware of modern computers is an incarnation of this necessity.

Metaphor helps a human being to breathe in this rarefied atmosphere of Gods.

"It Is Still Love"

The Siege
by Clara Park,
Atlantic Monthly Press, 1982, 328 pp.

In 1958, in a small American city, a fourth child was born to Clara and David Park. They named the girl Jessy. By the time she was two, her retarded development and introvertedness began to worry her parents. In a while the diagnosis was confirmed: childhood autism.

The syndrome of early childhood autism (from the Greek $\alpha\dot{\upsilon}\tau\acute{o}\sigma$ – self) was described in the early forties by the American psychiatrist L. Kanner. The symptoms are evident by the age two to four years, and involve above all the absence of interaction with other people, including the mother, and an extreme isolation from the outside world. The development of speech can be sharply retarded, as it happened to Jessy. Even if it is not retarded, speech is anomalous: it is not directed towards communication. An autistic child cannot endure a change in his or her habitual surroundings and regular daily routine, but has a wonderful mechanical memory and good physical health. All of the child's actions are dominated by a shutting out of other people.

This is how it appears: " [...] a tiny golden child on hands and knees, circling round and round a spot on the floor in mysterious, self-absorbed delight. She does not look up, though she is smiling and laughing; she does not call our attention to the mysterious object of her pleasure. She does not see us at all. She and the spot are all there is, and though she is eighteen months old, an age for touching, tasting, pointing, pushing, exploring, she is doing none of these. She does not walk, or crawl up stairs, or pull herself to her feet to reach for objects. She does not *want* any objects." (p. 3.)

These are the external diagnostic signs. Nobody knows the underlying mechanism of childhood autism. Jessy's childhood occurred at a time when familiarity with Kanner's syndrome was rare, even among professionals, and the practical recommendations for parents and teachers were just being formulated and were not readily available.

David Park was teaching physics at a small college, and Clara had graduated from a university. Before Jessy's birth she intended to pursue a professional career after the children had grown. Instead, the condition afflicting her youngest child forced her to renounce her previous plans. After the seriousness of the anomaly became clear, Jessy's parents decided not to send the girl to an institution, but rather to raise her in the family (and later in a special school as well.)

First published in Russian, *Priroda*, 1987, no. 4, 118–120. Translated by the author.

Clara Park belonged to the generation of "the mothers of the forties and fifties for whom Dr. Spock had replaced the conventional wisdom." (p. 14.) That meant that she and her friends not only watched the development of their children and shared the experience with each other, but they also read books on children's psychology, thought about them, and in general invested the full strength of their hearts and minds—I stress this, hearts and minds—into the raising of children. The rebellious generation of the sixties was brought up by these mothers as well as their own epoch.

The long and exhausting labor of love that Clara Park began in combating Jessy's autism, about which she has written a book six or seven years later, Clara called a siege. To understand the meaning of this word, we must glance at Jessy through the eyes of her mother, and imagine the embryonic, bundled up, budlike consciousness of a little person, that has hidden itself behind invisible fortress walls, self-sufficient, unapproachable by human words, unyielding to the enticements of the world and its sounds, closed off even from those closest to her.

Clara Park's daily and attentive observations over many years are invaluable for the study and therapy of children's autism. The book paints an excellent picture of the psyche of autism, whose primary characteristic is a motivational defect—the child acts as though he or she does not want to grow up. Therein lies the recurring metaphor: Jessy is the magical child from the Land of the Babes in Irish folklore. Such a view of Jessy turned out to be pragmatically valuable in formulating a strategy for combating the autistic mentality. However, Clara Park fully realizes (and affirms this understanding with numerous observations and interpretations), that the affection is a multifaceted one, and it touches all areas of the psyche.

Based on his large professional experience, Dr. V. E. Kagan in his book, *Children's Autism*, proposes that Kanner's syndrome forces us to accept human communication as a separate function of psychics. Conceivably, it is aided by specialized mechanisms of perception, such as, for example, the visual recognition of people's faces, which can fail as an isolated function in a rare form of visual agnosia (the disruption of the process of recognition)—prosopagnosia.

When she was about three years old, Jessy learned to put together jigsaw puzzles. In putting together a puzzle, children are usually guided by a picture which they try to reconstruct. But Jessy was so attuned to abstract forms that she was able to put the jigsaw together upside down. On the other hand, the perception of the picture itself was greatly diminished, or was completely absent. Characteristically, she was unable to solve a simple five piece jigsaw puzzle in which the last piece was a sun with eyes. Round, and almost symmetric, it gave no clue as to the proper placement by its form: it was necessary for the eyes to be at the top. Jessy could not comprehend this: "Eyes—faces—were simply not within her scheme of relevance" (p. 60.)

Many pages of the book provide excellent material for a discussion of the disruption of the functional interrelationship between the two cerebral hemispheres in children with Kanner's syndrome. There appears to be a general reduction in the activity of Jessy's dominant (speech, left) hemisphere and a simultaneous hyperactivation of the subdominant hemisphere. For example, during the sharply slowed development of speech, beginning at about age four, Jessy began to develop a compensatory utilization of melody as a replacement of words.

The song "Ring around a rosy", accompanying a round dance game, subsequently was used as the name of this game, of the picture of the round dance in a book, of a wreath, and finally of a circle "and a cluster of ideas around it, functioning far more reliably than any of her actual words". This melodic vocabulary consisting of *leitmotifs* as Clara calls them kept extending. "We noticed that though she now sang many songs freely, she never sang her leitmotifs at random or for their own sake as songs. Nor did she sing them musically, like the others, but rapidly, schematically, *functionally*—only just well enough for them to do their job of communication." (p. 84.)

By five years of age, Jessy's vocabulary was limited to only thirty or forty isolated words, but eventually it began to increase rapidly—according to Kanner, this was a definitive sign of a favorable prognosis. During her sixth year, she started to appropriate new words at the rate of a normal two year old, but did not begin to speak at the same rate as a normal two year old. The acquisition of a new lexicon, as well as semantical and syntactical peculiarities of her speech were very different from the norm. Clara Park's searching observations indicate that Jessy was learning her native language as a foreigner, thus adding additional evidence that her psyche was predominantly right-hemispherical. (Some studies seem to show that the acquisition of the second language in the early stages of life proceeds with a significant participation of the subdominant hemisphere.)

Restricted ability of contact with others imposed certain characteristics to Jessy's speaking competence. She learned and properly used words such as "oak", "elm", and "maple" without any difficulty. However, words such as "sister", "grandmother", "friends" and "stranger" were semantically inaccessible to her at the age of five, and the last words—even at the age of seven. This was not due to an inability to understand the abstract in general, since Jessy was able to correctly distinguish and use the geometric notions of "triangle", "rectangle", and so on up to "octagon". What was incomprehensible was a group of abstractions dealing with human relationships. The classical symptom of autism, the employment of "I" for others and "you" for self, is an example of such a lack of understanding. A normal child quickly passes through this development stage, having realized that pronouns change their meaning when the speaker changes, but the autistic consciousness lingers on this concept. The meaning of the pronouns "he", "she" and "they", Jessy understood only by the age of eight, and even then with great difficulty.

Jessy's other speech peculiarities, heard by her mother's attentive ears, require more subtle interpretation. Here is one of the many observations.

"Small children say "bad" with every gradation of fear and fury; [Jessy] now says "bad" too. But she says it with serene pleasure, to set a phenomenon in its proper category: "Bad can," she says as she collects beer cans from the beach. "Bad dog," she remarks, surveying an overturned garbage can. [Jessy] does not care for dogs. If one comes too close she clings to me; if it jumps up she whimpers. But it would never occur to her to verbalize her emotion. She would not say "bad dog" then." (pp. 211–212.)

Such an inability to verbalize one's emotions is tied with a general deficiency of self-identification, the process of formation of one's own "I". With almost experimental exactness, autism indicates that one can recognize self only if one can recognize other selves.

One last note concerning the frequently increased aptitude of an autistic child in the sphere of elementary mathematics, which becomes evident if it is encouraged by education. An example from Jessy's life is characteristic in this area as well. Around the age of seven and a half, Jessy's mother began to teach her addition: 1+1=2, 2+1=3. Clara did not explain her "zero"; this difficult concept did not appear until late in the history of civilization, and Clara decided that if the ancient Greeks could do without zero, so could Jessy for the time being. It turned out, however, that Jessy had already heard about zero, and she objected: "No zero!" She wanted 0+1=1 and I supplied it. Then, "Oh, we forgot! Zero plus zero equals zero!" (p. 241.)

In contemporary psychiatry, the terms "autism" and "autistic behavior" are also used in a more general sense, not only to refer to psychological disorders, but to characterize some features of a normal psyche as well. When Clara Park returned to teaching, she found that observations of Jessy opened her eyes to a great deal about her students—people with normal abilities, who learned to read and write, and efficiently carried out their various duties, but were at times so similar to Jessy in the system of inner impediments which hindered their capability to work and live.

A contemporary Russian writer contemptuously called his book, *An upbringing according to Dr. Spock*, an invective against foreign and uncomfortable habits of our time. Such a label choice is both coincidental, and characteristic, and deserves our compassion. An individual's autism finds its parallels in familial, confessional, and national autisms. Invisible fortress walls divide the world into separate prison cells; too many of our deeds pile more rocks onto these walls; everyone risks to end in a solitary confinement. "Strange [Jessy] is so like us ." (p. 274.)

After the publication of the first edition in 1967, Clara Park's book came out in several languages. We are reviewing the 1982 edition, with the additional epilogue, "Fifteen years later". Jessy is now 23 years old; Clara states, with pride, that she is working, owns a bank account, and that she will soon be paying taxes, just like any other competent citizen. Moreover, she became a successful painter, and her works are exhibited and purchased. A black and white reproduction of "A heater in Valerie's bath"—pop-art—is pasted into the book and is described in several expressive sentences; we can imagine the intense glowing acrylic colors of this work.

A happy end?

Of course not. Life histories do not end while their protagonists are still alive, only the story ends. Jessy did not become a healthy person. The life she leads differs greatly from the lives her brother and sister lead. A happy end?

Let's listen one last time to Clara Park.

"Let me say simply and straight out that simple knowledge the whole world knows. I breathe like everyone else my century's thin, faithless air, and I do not want to be sentimental. But the blackest sentimentality of all is that *trahison des clercs* which will not recognize the good it has been given to understand because it is too simple. So, then: this experience we did not choose, which we would have given everything to avoid, has made us different, has made us better. Through it we have learned the lesson that no one studies willingly, the hard, slow lesson of Sophocles and Shakespeare—that one grows by suffering. And that too is Jessy's gift. I write now what fifteen years past I would still not have thought possible to write: that if today I were given the choice, to accept the experience, with everything that it entails, or to refuse the bitter largesse, I would have to stretch out my hands—because out of it has come, for all of us, an unimagined life. And I will not change the last word of the story. It is still love." (p. 320.)