Contents

Publisher's Note	ix
Editor's Preface	xi
Introduction	xvii
Chapter 1. Geometry and three-manifolds	1
Chapter 2. Elliptic and hyperbolic geometry	7
2.1. The Poincaré disk model	9
2.2. The southern hemisphere	10
2.3. The upper half-space model	11
2.4. The projective model	11
2.5. The sphere of imaginary radius	14
2.6. Trigonometry	15
Chapter 3. Geometric structures on manifolds	23
3.1. A hyperbolic structure on the figure-eight knot complement	25
3.2. A hyperbolic manifold with geodesic boundary	26
3.3. The Whitehead link complement	27
3.4. The Borromean rings complement	28
3.5. The developing map	29
3.8. Horospheres	32
3.9. Hyperbolic surfaces obtained from ideal triangles	34
3.10. Hyperbolic manifolds obtained by gluing ideal polyhedra	36
Chapter 4. Hyperbolic Dehn surgery	37
4.1. Ideal tetrahedra in H^3	37
4.2. Gluing consistency conditions	40
4.3. Hyperbolic structure on the figure-eight knot complement	41
4.4. The completion of hyperbolic 3-manifolds obtained from ideal polyhedra	44
4.5. The generalized Dehn surgery invariant	45
4.6. Dehn surgery on the figure-eight knot	46
4.8. Degeneration of hyperbolic structures	49
4.10. Incompressible surfaces in the figure-eight knot complement	57
Chapter 5. Flexibility and rigidity of geometric structures	71
5.1. Deformations of geometric structure	71

vi CONTENTS

5.2.	A crude dimension count	72
5.3.	Teichmüller space	74
5.4.	Special algebraic properties of groups of isometries of H^3	79
5.5.	The dimension of the deformation space of a hyperbolic three-manifold	82
5.7.	Mostow's Theorem	86
5.8.	Generalized Dehn surgery and hyperbolic structures	87
5.9.	A Proof of Mostow's Theorem	90
5.10.	A decomposition of complete hyperbolic manifolds	97
5.11.	Complete hyperbolic manifolds with bounded volume	99
5.12.	Jørgensen's Theorem	102
Chapte	r 6. Gromov's invariant and the volume of a hyperbolic manifold	105
6.1.	Gromov's invariant	105
6.3.	Gromov's proof of Mostow's Theorem	111
6.4.	Strict version of Gromov's Theorem	111
6.5.	Manifolds with boundary	115
6.6.	Ordinals	119
	Commensurability	120
6.8.	Some examples	123
	r 7. Computation of volume	135
7.1.	The Lobachevsky function $\pi(\theta)$	135
	r J	138
	Some manifolds	143
	Arithmetic examples	144
Refe	rences	146
	r 8. Kleinian groups	149
8.1.	The limit set	149
8.2.	The domain of discontinuity	151
8.3.	Convex hyperbolic manifolds	154
8.4.	Geometrically finite groups	157
8.5.	The geometry of the boundary of the convex hull	161
8.6.	Measuring laminations	165
8.7.	Quasi-Fuchsian groups	167
8.8.	Uncrumpled surfaces	174
8.9.	The structure of geodesic laminations: train tracks	178
8.10.	<u> </u>	182
8.11.	•	188
8.12.	Harmonic functions and ergodicity	191
Chapte	ŭ .	195
9.1.	Limits of discrete groups	195
9.2.	Geometric tameness	198
9.3.	The ending of an end	202
9.4.	Taming the topology of an end	208

CONTENTS vii

9.5.	Interpolating negatively curved surfaces	210
9.6.	Strong convergence from algebraic convergence	225
	Realizations of geodesic laminations for surface groups with extra cusps, with	
	a digression on stereographic coordinates	228
9.9.	Ergodicity of the geodesic flow	242
NOTE		249
Chapter	11. Deforming Kleinian manifolds by homeomorphisms of the sphere at	
	infinity	251
11.1.	Extensions of vector fields	251
Chapter	13. Orbifolds	261
13.1.	Some examples of quotient spaces	261
13.2.	Basic definitions	264
13.3.	Two-dimensional orbifolds	271
13.4.	Fibrations	279
13.5.	Tetrahedral orbifolds	284
13.6.	Andreev's theorem and generalizations	290
13.7.	Constructing patterns of circles	297
13.8.	A geometric compactification for the Teichmüller spaces of polygonal	
	orbifolds	305
13.9.	A geometric compactification for the deformation spaces of certain Kleinian	
	groups.	309
Index		313