

# Index

- 17-gon, 16, 161-162
- 18-gon, 167
- 20-80-80 triangle, 167
- 20-gon, 174
- 24-gon, 162-164
- 26-gon, 168
- 48-gon, 162-164
- 60-gon, 169
- 96-gon, 162-164
- 257-gon, 16, 166
- 384-gon, 165
- 3072-gon, 165
- 12288-gon, 165
- 24576-gon, 165
- 65537-gon, 16, 166-167
- $3 \cdot 2^{17}$ -gon, 165
- $60 \cdot 2^{33}$ -gon, 165
- $2^{62}$ -gon, 165
- almost isosceles Pythagorean triples, 206
- alternating series, 85
- angles of a polygon, 2
- aperiodic tiling, 152
- approximate
  - angle trisection, 144
  - drawings of polygons, 111, 154
- approximation
  - by triangulation, 110
  - to  $\pi$ , 142, 162-165, 178
- Archimedean tiling, 20
  - approximation to  $\pi$ , 142, 162, 164
- area of a
  - cyclic hexagon, 71
  - decagon, 148, 149
  - dodecagon, 156, 158, 159, 173
  - heptagon, 105
  - hexagon, 64, 71, 80
  - hexagram, 86
  - median triangle, 78
  - octagon, 123, 125
  - pentagon, 30, 32
  - pentagram, 49
  - polygon, 8
  - star octagon, 122
  - star polygon, 18, 19
  - tangential polygon, 191
- arithmetic progression, 201
- assigning numbers to points in the plane, 67, 68

- Babylonian approximation  
     to  $\pi$ , 100, 101  
 baseball's pentagon, 42, 43  
 benzene molecule, 70  
 bicentric polygon, 4, 194,  
     195  
 buckminsterfullerene, 52  
  
 Cairo pentagonal tiling, 25,  
     44  
 calissons, 68, 69  
 carpenter's square, 81, 82,  
     112, 144  
 Cassini's identity, 93  
 Catalan numbers, 66  
 centered polygonal number,  
     197-199, 205-206  
 centrally symmetric convex  
     polygon, 183  
 centroid parahexagon, 78  
 circumcenter of a cyclic  
     polygon, 4  
 circumcircle of a cyclic  
     polygon, 4  
 circumradius of a  
     cyclic polygon, 4, 74  
     regular polygon, 8  
     regular star polygon,  
         19  
 clipped rectangle, 57, 215,  
     216  
 compass, 14  
     rose, 133  
 complex polygon, 3, 29  
 compound polygon, 3  
 concave polygon, 3, 29  
 constructible regular poly-  
     gon, 15, 141  
  
 construction of equidiagonal  
     irregular pentagon, 41  
     of equilateral irregular  
         pentagon, 40  
     with straightedge and  
         compass, 14  
 convex parahexagon, 81  
     polygon, 2, 29  
 cordovan proportion, 126  
     triangles, 126  
 counting calissons, 68  
 Crockett Johnson, 117  
 crossed polygon, 29  
 cyclic hexagon, 71-75  
     polygon, 4, 186-190  
  
 decagon, 148-153  
 decagons  
     in architecture, 153  
     in polyhedra, 152, 153  
     in space, 152  
     in tessellations, 152  
 density of a star polygon, 17  
 diacosipentacontaheptagon,  
     167  
 diagonals of a regular  
     decagon, 150  
     dodecagon, 157, 158  
     heptagon, 105-107  
     hexagon, 66, 72  
     nonagon, 146  
     octagon, 123-125  
     pentagon, 31, 42  
     polygon, 11-14  
 diagonals of convex poly-  
     gons, 5-7  
 dodecagon, 155-160  
 dodecagonal coins, 157

- dodecagons
  - in architecture, 160
  - in tessellations, 160
- dodecahedron, 52, 55
- domino, 83
- drawing a pentagon, 14, 30, 35-37
  - a tesseract, 131
- drawing a regular
  - decagon, 149
  - dodecagon, 155
  - hendecagon, 154, 155
  - heptagon, 109-112
  - hexacontagon, 169
  - hexagon, 63, 64
  - nonagon, 143
  - octagon, 122
- Drummond's approximation, 154
- dual tiling, 23-25
- Dürer's approximate drawings, 110, 145, 154
  - solid, 53
- edge-to-edge irreducible
  - dissection, 183
  - tiling, 20
- enneagon, 143
- equable polygon, 196, 242
- equiangular polygon, 4, 189, 191, 194
- equilateral irregular pentagonal tiling, 44
  - pentagon, 4, 194
- Erector set polygon models, 39
- Euclid's *Elements*, ix, 7, 14, 15, 35, 202
  - even perfect number, 202
  - exterior angles, 26, 27, 214
- Fermat prime, 15, 16
- Fibonacci numbers, 92, 93
- figurate number, 197
- fire hydrant pentagon, 34
- floret pentagonal tiling, 25, 45
- folding a regular pentagon, 38, 39
- friends and strangers theorem, 93, 94
- Fuhrmann's theorem, 73
- Gauss-Wantzel theorem, 15, 103, 141
- geoboard, 177
- geometric mean, 10
  - series, 125
- Gleason's construction, 111, 112
- golden bee, 86
  - gnomon, 33, 50
  - ratio, 31, 32, 48-50, 54, 58, 86
  - triangle, 33, 50
- golygon, 182
- great heptagram, 115
- harmonic mean, 10
- hendecagon, 151-155
- hendecagons in architecture, 141, 161, 162
- heptadecagon, 141, 161, 162
- heptagon, 103-117
- heptagonal coins, 104
  - tiling, 114
  - triangle, 106-109

- heptagons in architecture, 116, 117
- heptagram, 115
- Heron's formula, 176
- Hex (game), 74, 75
  - number, 205
- hexacontagon, 169
- hexagon, 61-101
- hexagon-octagon approximation of  $\pi$ , 125
- hexagonal number, 201-202
  - proofs, 91-93
  - section of cube, 77
  - tilings, 75-77
  - truncated trapezohedron, 52
  - zonogon, 115
- hexagons
  - in architecture, 95
  - in polyhedra, 64, 65
  - in semiregular tilings, 76
- hexagram, 62, 86-90
- hexayurt, 96
- Hirano's construction, 36, 37
- hole-free rectilinear polygon, 181
- honeycomb, 63
- hypercube, 131
- icosagon, 174
- incircle of a tangential polygon, 4
- inradius of a
  - regular polygon, 8
  - regular star polygon, 19
  - tangential polygon, 4
- interior of a polygon, 2
- irrationality of the golden ratio, 32
- irreducible dissection, 183-184
- irregular reptiles (irreptiles), 85
- isoperimetric inequality, 9
  - theorem, 7, 9
- James Webb Space Telescope, 94
- Japanese theorem, 190
- Johnson's theorem, 80
- Koch snowflakes, 97
- lattice heptagon, 180
  - hexagon, 180
  - octagon, 180
  - parahexagon, 180
  - pentagon, 179
  - point, 175
  - polygon, 175-180
  - square, 175
  - triangle, 176
- Lloyd's puzzles, 59, 60, 119
- L-hexagon, 62, 181
- L-polyomino, 82, 86, 98, 181
- L-tetromino, 84
- L-tromino, 84, 85
- Lucas numbers, 49
- lune, 90
- magic hexagram, 88
- Marion Walter's theorem, 99, 222, 223
- Meccano polygons, 39
- metallic ratios, 126

- monohedral tiling, 20, 23
- Morrie's law, 146, 170
- multi-polygonal number, 203
- $n$ -gon, 2
- $n^{\text{th}}$  ordinary  $k$ -gonal number, 200
- neusis construction, 112
- nonagon, 143-148
- nonagons
  - in architecture, 148
  - in tilings, 147
- octagon, 121-140
- octagonal objects, 123
  - tiling, 127
- octagons and parallelograms, 128, 129
- octagons
  - in architecture, 133-137
  - in art, 137
  - in space, 137
  - in urban design, 136
- octagram, 131
  - as a compass rose, 133
- ordinary polygonal number, 197, 199, 200-205
- ordinary triangular number, 200
- orthogonal polygon, 175, 181-183
- panoply, x
- parallel pentagon, 39
- parahexagon, 2, 61, 62, 77, 81
- parallelogon, 81
- Pascal line, 89
- Pascal's hexagon theorem, 89
  - triangle, 203
- Penrose aperiodic tiling, 152
- pentacle, 48
- pentadecagon, 169
- pentagon, 29-60
  - trigonometry, 31, 32
- pentagonal logos, 37
  - number, 201, 204
  - rank, 204
  - tilings, 43-48
- pentagon-hexagon-decagon identity, 151
- pentagons
  - in architecture, 56, 57
  - in nature, 30
  - in space, 52-54
- pentagram, 29, 48-52
  - in art and architecture, 50
  - on flags, 51, 52
- pentomino, 83
- perfect number, 202
  - tiling, 85
- perimeter of a polygon, 9
- Petrie polygon, 63
- pi ( $\pi$ ), approximations to, 1, 131, 142, 162, 165
  - Archimedean value, 178
  - Babylonian value, 178
- Pick's theorem, 177
- Pierpont prime, 112
- polygon, 1-27
  - bicentric, 4, 194, 195
  - complex, 3, 29
  - compound, 3

- polygon,  
   concave, 3, 29  
   convex, 2, 29  
   crossed, 29  
   cyclic, 4, 186-190  
   equable, 196, 242  
   equiangular, 4, 189, 191,  
     194  
   equilateral, 4, 194  
   lattice, 175-180  
   rectilinear, 175, 181-183  
   regular, 4, 7-14  
   reptile, 54, 85, 98  
   simple, 2  
   skew, 4  
   star, 3, 16-19  
   tangential, 4, 175, 190-  
     194  
   unicursal, 2  
 polygonal floor tiling, 20  
   number, 197-210  
   tiling, 1, 2, 19-23  
 polygons, classification, 2-5  
   in Moorish art, 1, 2  
 polyominoes, 82  
 Poncelet's closure theorem,  
   195  
 Ptolemy's construction, 35  
   theorem, 13  
 Putnam octagons, 137, 139,  
   230, 232, 233  
   pentagon, 58, 216  
 Pythagorean theorem, 91, 92  
   triple, 206  
  
 rectilinear hexagon, 181  
   octagon, 181  
   polygon, 175, 181-183  
  
 reflex angle, 3, 18  
 regular decagon, 148-150  
   dodecagon, 155-158  
   hendecagon, 153  
   heptagon, 105-107  
   hexagon, 63-70  
     and lunes, 90  
   nonagon, 143, 144  
   octagon, 122-128  
   pentagon, 30-34  
   polygon, 4, 7-14  
   tiling, 20, 21  
 reptile (polygon), 54, 85, 98  
 Reuleaux polygon, 104  
 Rhind papyrus, 130, 131  
 Richmond's construction,  
   35, 36  
  
 sangaku, 36, 190  
 self-intersecting polygon, 2,  
   3  
 semiperimeter of a regular  
   polygon, 8  
 semiregular tiling, 20-22  
 serial isogon of  $90^\circ$ , 182  
   of  $60^\circ$ , 183  
 silver ratio, 125  
   rectangle, 126  
 simple lattice polygon, 177  
   polygon, 2  
 skew polygon, 4  
 snowflake, 97  
 sphinx concave polygon, 54  
 square centered square  
   number, 206  
   triangular number, 203  
 star, 18  
   dodecagon, 2

- star figure, 17, 86
  - heptagon, 115, 116
  - hexagon, 2
  - number, 207, 208
  - of David, 86
  - of Lakshmi, 131, 132
  - polygon, 3, 16-19
- Statue of Liberty, 155
- stellation of a polygon, 17
- straightedge, 14
  
- tangential polygon, 4, 175, 190-194
- tessellation, 19-23
- tesseract, 131
- tetromino, 83
- tiling, 19-23
  - monohedral, 20
  - regular, 20
  - semiregular, 20
  - uniform, 20
- trapezoidal number, 201, 207
- tromino, 83
  
- truncated cube, 133
  - cuboctahedron, 134
  - dodecahedron, 152, 153
  - icosahedron, 52, 65
  - octahedron, 65
  - rhombhedron, 53
  - tetrahedron, 65
  
- undecagon, 153
- unicursal hexagon, 88-89
  - hexagram, 88
  - polygon, 2
- uniform tiling, 20, 147
- unmarked straightedge, 14
  
- Van Schooten's theorem, 67
- vertex angle, 8, 19
- Viviani's theorem, 11
- Voronoi diagram, 23-25
  
- Wallace-Bolyai-Gerwien theorem, 159
  
- zonogon, 175, 183-186