Preface

Geometry is the archetype of the beauty of the world.

Johannes Kepler

Geometry is nothing at all, if not a branch of art.

Julian Lowell Coolidge

Beauty is geometry.

J. K. Rowling

To be a well-educated person, the study of geometry should be included in one's academic background. And some of the most enchanting objects in elementary geometry are the *polygons*, figures constructed from two basic geometric concepts, line segments and angles. They are prime examples of great beauty created from the simplest of mathematical objects, and are the subject of this Panoply.

What is a *Panoply*? Dictionary definitions include *a splendid or striking array or arrangement* and *a complete or impressive collection or display of something*. The objects in our Panoply are, for the most part, planar polygons with five or more sides. The restriction to five or more sides is because entire books can be and have been written about triangles (e.g., several books in Euclid's *Elements*) and quadrilaterals (e.g., the 2020 AMS-MAA book *A Cornucopia of Quadrilaterals*). But books devoted to polygons with five or more sides seem to be rather rare.

Polygons come in a variety of shapes and possess a multitude of different properties, as witnessed by the many adjectives used to describe them: *simple, complex, concave, convex, compound, equiangular, equilateral, regular, skew, star, cyclic, tangential, bicentric, rectilinear, lattice, equable*, etc. In addition we have the adjective *polygonal*, as in *polygonal numbers*, positive integers that enumerate sets of objects in polygonal patterns, numbers that have been studied since the time of the ancient Greek mathematicians. All of these types of polygons are present in our Panoply, and in many instances illustrated with examples from art and architecture.

A Panoply of Polygons consists of eight chapters. In the first chapter we present some basic facts about convex polygons and all regular polygons and regular star polygons, along with some mathematical tools applicable to polygons in subsequent chapters (e.g., Ptolemy's theorem to study cyclic polygons). In this chapter we also discuss the Gauss-Wantzel theorem and various methods for drawing regular polygons, and the regular polygons that appear in various regular and semiregular tilings of the plane.

Each of the next four chapters is devoted to a particular type of polygon. In Chapter 2 we present the properties of regular and irregular pentagons, including the relationship with the golden ratio, equidiagonal and parallel pentagons, pentagrams, and monohedral tilings of the plane with convex pentagons. Chapter 3 is devoted to hexagons, where we have not only regular hexagons and hexagrams, but also irregular ones such as parahexagons and certain polyominoes. In Chapter 4 we study heptagons and present some of the remarkable properties of the heptagonal triangle, whose sides are the two unequal diagonals and an edge of a regular heptagon. In addition we study the two regular star heptagons and present several ways to draw approximations of a regular heptagon. Chapter 5 presents octagons and octagrams in a manner analogous to the treatment of the polygons in the previous three chapters.

In Chapter 6 we have a selection of polygons with nine or more sides with some distinguishing properties, e.g., nonagons, decagons, and dodecagons. Others are included for their roles in the history of mathematics, such as the 17-gon and the 96-gon. Chapter 7 presents some classes of polygons sharing a common characteristic or property, such as lattice polygons, rectilinear polygons, and cyclic and tangential polygons. In Chapter 8 we conclude with a brief study of polygonal numbers, both ordinary and centered, and their relationships. Each chapter includes a set of exercises we call Challenges, with solutions to all the Challenges following Chapter 8. The book concludes with a bibliography and a complete index.

A Panoply of Polygons is not a textbook, although it can be used as a supplement to a high school or college geometry course. It can also be used as a source for group projects or extra-credit assignments. But more importantly, we believe it will be of interest to anyone who loves geometry.

A note about our notation: We use labels such as AB or a both for a line segment and for its length; the context suffices to indicate which. Similarly we use A or θ for both an angle and its measure (A may also be a vertex of a polygon). We use both degrees and radians for angle measurement; whichever is more convenient in a particular situation. The symbol ■ marks the end of a proof, while □ marks the end of an example.

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Claudi Alsina
Universitat Politècnica de Catalunya
Barcelona, Spain

Roger B. Nelsen
Lewis & Clark College
Portland, Oregon