

CONTENTS

VOLUME 1

Foreword	V
International Congresses of Mathematicians	1
Fields medalists and IMU prize winners	3
Opening greetings by the IMU President	5
Closing remarks by the IMU President	9
Status report for the IMU	11
Photographs	21

THE WORK OF THE FIELDS MEDALISTS AND THE IMU PRIZE WINNERS

Martin Hairer, The work of Hugo Duminil-Copin	26
Gil Kalai, The work of June Huh	50
Kannan Soundararajan, The work of James Maynard	66
Henry Cohn, The work of Maryna Viazovska	82
Ran Raz, The work of Mark Braverman	106
Henri Darmon, The work of Barry Mazur	118
Rupert L. Frank, The work of Elliott Lieb	142
Tadashi Tokieda, Nikolai Andreev and the art of mathematical animation and model-building	160

PRIZE LECTURES

Hugo Duminil-Copin, 100 years of the (critical) Ising model on the hypercubic lattice **164**

June Huh, Combinatorics and Hodge theory **212**

James Maynard, Counting primes **240**

Maryna Viazovska, On discrete Fourier uniqueness sets in Euclidean space **270**

Mark Braverman, Communication and information complexity **284**

Nikolai Andreev, Popularization of math: sketches of Russian projects and traditions **322**

Marie-France Vignéras, Representations of p -adic groups over commutative rings **332**

POPULAR SCIENTIFIC EXPOSITIONS

Andrei Okounkov, The Ising model in our dimension and our times **376**

Andrei Okounkov, Combinatorial geometry takes the lead **414**

Andrei Okounkov, Rhymes in primes **460**

Andrei Okounkov, The magic of 8 and 24 **492**

SUMMARIES OF PRIZE WINNERS' WORK

Allyn Jackson, 2022 Abacus Medal: Mark Braverman **548**

Allyn Jackson, 2022 Chern Medal: Barry Mazur **554**

Allyn Jackson, 2022 Gauss Prize: Elliott H. Lieb **560**

Allyn Jackson, 2022 Leelavati Prize: Nikolai Andreev **566**

List of contributors **571**

VOLUME 2

SPECIAL PLENARY LECTURES

Kevin Buzzard, What is the point of computers? A question for pure mathematicians **578**

Frank Calegari, Reciprocity in the Langlands program since Fermat's Last Theorem **610**

Frans Pretorius, A survey of gravitational waves **652**

PLENARY LECTURES

Mladen Bestvina, Groups acting on hyperbolic spaces—a survey **678**

Bhargav Bhatt, Algebraic geometry in mixed characteristic	712
Thierry Bodineau, Isabelle Gallagher, Laure Saint-Raymond, Sergio Simonella, Dynamics of dilute gases: a statistical approach	750
Alexander Braverman, David Kazhdan, Automorphic functions on moduli spaces of bundles on curves over local fields: a survey	796
Tobias Holck Colding, Evolution of form and shape	826
Camillo De Lellis, The regularity theory for the area functional (in geometric mea- sure theory)	872
Weinan E, A mathematical perspective of machine learning	914
Craig Gentry, Homomorphic encryption: a mathematical survey	956
Alice Guionnet, Rare events in random matrix theory	1008
Larry Guth, Decoupling estimates in Fourier analysis	1054
Svetlana Jitomirskaya, One-dimensional quasiperiodic operators: global theory, dual- ity, and sharp analysis of small denominators	1090
Igor Krichever, Abelian pole systems and Riemann–Schottky-type problems	1122
Alexander Kuznetsov, Semiorthogonal decompositions in families	1154
Scott Sheffield, What is a random surface?	1202
Kannan Soundararajan, The distribution of values of zeta and L-functions	1260
Catharina Stroppel, Categorification: tangle invariants and TQFTs	1312
Michel Van den Bergh, Noncommutative crepant resolutions, an overview	1354
Avi Wigderson, Interactions of computational complexity theory and mathematics	1392
List of contributors	1433

VOLUME 3

1. LOGIC

Gal Binyamini, Dmitry Novikov, Tameness in geometry and arithmetic: beyond o-minimality	1440
Natasha Dobrinen, Ramsey theory of homogeneous structures: current trends and open problems	1462
Andrew S. Marks, Measurable graph combinatorics	1488
Keita Yokoyama, The Paris–Harrington principle and second-order arithmetic— bridging the finite and infinite Ramsey theorem	1504

Dmitriy Zhuk, Constraint satisfaction problem: what makes the problem easy **1530**

2. ALGEBRA

Pierre-Emmanuel Caprace, George A. Willis, A totally disconnected invitation to locally compact groups **1554**

Neena Gupta, The Zariski cancellation problem and related problems in affine algebraic geometry **1578**

Syu Kato, The formal model of semi-infinite flag manifolds **1600**

Michael J. Larsen, Character estimates for finite simple groups and applications . . **1624**

Amnon Neeman, Finite approximations as a tool for studying triangulated categories **1636**

Irena Peeva, Syzygies over a polynomial ring **1660**

3. NUMBER THEORY – SPECIAL LECTURE

Joseph H. Silverman, Survey lecture on arithmetic dynamics **1682**

3. NUMBER THEORY

Raphaël Beuzart-Plessis, Relative trace formulae and the Gan–Gross–Prasad conjectures **1712**

Ana Caraiani, The cohomology of Shimura varieties with torsion coefficients **1744**

Samit Dasgupta, Mahesh Kakde, On the Brumer–Stark conjecture and refinements **1768**

Alexander Gamburd, Arithmetic and dynamics on varieties of Markoff type **1800**

Philipp Habegger, The number of rational points on a curve of genus at least two . **1838**

Atsushi Ichino, Theta lifting and Langlands functoriality **1870**

Dimitris Koukoulopoulos, Rational approximations of irrational numbers **1894**

David Loeffler, Sarah Livia Zerbes, Euler systems and the Bloch–Kato conjecture for automorphic Galois representations **1918**

Lillian B. Pierce, Counting problems: class groups, primes, and number fields **1940**

Sug Woo Shin, Points on Shimura varieties modulo primes **1966**

Ye Tian, The congruent number problem and elliptic curves **1990**

Xinwen Zhu, Arithmetic and geometric Langlands program **2012**

4. ALGEBRAIC AND COMPLEX GEOMETRY – SPECIAL LECTURE

Marc Levine, Motivic cohomology **2048**

4. ALGEBRAIC AND COMPLEX GEOMETRY

Mina Aganagic, Homological knot invariants from mirror symmetry	2108
Aravind Asok, Jean Fasel, Vector bundles on algebraic varieties	2146
Arend Bayer, Emanuele Macrì, The unreasonable effectiveness of wall-crossing in algebraic geometry	2172
Vincent Delecroix, Élise Goujard, Peter Zograf, Anton Zorich, Counting lattice points in moduli spaces of quadratic differentials	2196
Alexander I. Efimov, K-theory of large categories	2212
Tamás Hausel, Enhanced mirror symmetry for Langlands dual Hitchin systems ...	2228
Bruno Klingler, Hodge theory, between algebraicity and transcendence	2250
Chi Li, Canonical Kähler metrics and stability of algebraic varieties	2286
Aaron Pixton, The double ramification cycle formula	2312
Yuri Prokhorov, Effective results in the three-dimensional minimal model program	2324
Olivier Wittenberg, Some aspects of rational points and rational curves	2346
List of contributors	2369

VOLUME 4

5. GEOMETRY – SPECIAL LECTURES

Bruce Kleiner, Developments in 3D Ricci flow since Perelman	2376
Richard Evan Schwartz, Survey lecture on billiards	2392

5. GEOMETRY

Richard H. Bamler, Some recent developments in Ricci flow	2432
Robert J. Berman, Emergent complex geometry	2456
Danny Calegari, Sausages	2484
Kai Cieliebak, Lagrange multiplier functionals and their applications in symplectic geometry and string topology	2504
Penka Georgieva, Real Gromov–Witten theory	2530
Hiroshi Iritani, Gamma classes and quantum cohomology	2552
Gang Liu, Kähler manifolds with curvature bounded below	2576
Kathryn Mann, Groups acting at infinity	2594

Mark McLean, Floer cohomology, singularities, and birational geometry	2616
Iskander A. Taimanov, Surfaces via spinors and soliton equations	2638
Lu Wang, Entropy in mean curvature flow	2656
Robert J. Young, Composing and decomposing surfaces and functions	2678
Xin Zhou, Mean curvature and variational theory	2696
Xiaohua Zhu, Kähler–Ricci flow on Fano manifolds	2718

6. TOPOLOGY

Jennifer Hom, Homology cobordism, knot concordance, and Heegaard Floer homology	2740
Daniel C. Isaksen, Guozhen Wang, Zhouli Xu, Stable homotopy groups of spheres and motivic homotopy theory	2768
Yi Liu, Surface automorphisms and finite covers	2792
Roman Mikhailov, Homotopy patterns in group theory	2806
Thomas Nikolaus, Frobenius homomorphisms in higher algebra	2826
Oscar Randal-Williams, Diffeomorphisms of discs	2856
Jacob Rasmussen, Floer homology of 3-manifolds with torus boundary	2880
Nathalie Wahl, Homological stability: a tool for computations	2904

7. LIE THEORY AND GENERALIZATIONS

Evgeny Feigin, PBW degenerations, quiver Grassmannians, and toric varieties	2930
Tasho Kaletha, Representations of reductive groups over local fields	2948
Joel Kamnitzer, Perfect bases in representation theory: three mountains and their springs	2976
Yiannis Sakellaridis, Spherical varieties, functoriality, and quantization	2998
Peng Shan, Categorification and applications	3038
Binyong Sun, Chen-Bo Zhu, Theta correspondence and the orbit method	3062
Weiqiang Wang, Quantum symmetric pairs	3080

8. ANALYSIS – SPECIAL LECTURE

Keith Ball, Convex geometry and its connections to harmonic analysis, functional analysis and probability theory	3104
--	-------------

8. ANALYSIS

Benôit Collins, Moment methods on compact groups: Weingarten calculus and its applications	3142
Mikael de la Salle, Analysis on simple Lie groups and lattices	3166
Xiumin Du, Weighted Fourier extension estimates and applications	3190
Cyril Houdayer, Noncommutative ergodic theory of higher rank lattices	3202
Malabika Pramanik, On some properties of sparse sets: a survey	3224
Gideon Schechtman, The number of closed ideals in the algebra of bounded operators on Lebesgue spaces	3250
Pablo Shmerkin, Slices and distances: on two problems of Furstenberg and Falconer	3266
Konstantin Tikhomirov, Quantitative invertibility of non-Hermitian random matrices	3292
Stuart White, Abstract classification theorems for amenable C^* -algebras	3314
Tianyi Zheng, Asymptotic behaviors of random walks on countable groups	3340
List of contributors	3367

VOLUME 5

9. DYNAMICS

Miklós Abért, On a curious problem and what it lead to	3374
Aaron Brown, Lattice subgroups acting on manifolds	3388
Jon Chaika, Barak Weiss, The horocycle flow on the moduli space of translation surfaces	3412
Mark F. Demers, Topological entropy and pressure for finite-horizon Sinai billiards	3432
Romain Dujardin, Geometric methods in holomorphic dynamics	3460
David Fisher, Rigidity, lattices, and invariant measures beyond homogeneous dynamics	3484
Mariusz Lemańczyk, Furstenberg disjointness, Ratner properties, and Sarnak’s conjecture	3508
Amir Mohammadi, Finitary analysis in homogeneous spaces	3530
Michela Procesi, Stability and recursive solutions in Hamiltonian PDEs	3552
Corinna Ulcigrai, Dynamics and “arithmetics” of higher genus surface flows	3576
Péter P. Varjú, Self-similar sets and measures on the line	3610

10. PARTIAL DIFFERENTIAL EQUATIONS

Tristan Buckmaster, Theodore D. Drivas, Steve Shkoller, Vlad Vicol, Formation and development of singularities for the compressible Euler equations	3636
Pierre Cardaliaguet, François Delarue, Selected topics in mean field games	3660
Semyon Dyatlov, Macroscopic limits of chaotic eigenfunctions	3704
Rita Ferreira, Irene Fonseca, Raghavendra Venkatraman, Variational homogenization: old and new	3724
Rupert L. Frank, Lieb–Thirring inequalities and other functional inequalities for orthonormal systems	3756
Alexandru D. Ionescu, Hao Jia, On the nonlinear stability of shear flows and vortices	3776
Mathieu Lewin, Mean-field limits for quantum systems and nonlinear Gibbs measures	3800
Kenji Nakanishi, Global dynamics around and away from solitons	3822
Alexander I. Nazarov, Variety of fractional Laplacians	3842
Galina Perelman, Formation of singularities in nonlinear dispersive PDEs	3854
Gabriella Tarantello, On the asymptotics for minimizers of Donaldson functional in Teichmüller theory	3880
Dongyi Wei, Zhifei Zhang, Hydrodynamic stability at high Reynolds number	3902

11. MATHEMATICAL PHYSICS – SPECIAL LECTURE

Peter Hintz, Gustav Holzegel, Recent progress in general relativity	3924
---	------

11. MATHEMATICAL PHYSICS

Roland Bauerschmidt, Tyler Helmuth, Spin systems with hyperbolic symmetry: a survey	3986
Federico Bonetto, Eric Carlen, Michael Loss, The Kac model: variations on a theme	4010
Søren Fournais, Jan Philip Solovej, On the energy of dilute Bose gases	4026
Alessandro Giuliani, Scaling limits and universality of Ising and dimer models ...	4040
Matthew B. Hastings, Gapped quantum systems: from higher-dimensional Lieb–Schultz–Mattis to the quantum Hall effect	4074
Karol Kajetan Kozłowski, Bootstrap approach to 1+1-dimensional integrable quantum field theories: the case of the sinh-Gordon model	4096
Jonathan Luk, Singularities in general relativity	4120

Yoshiko Ogata, Classification of gapped ground state phases in quantum spin systems	4142
List of contributors	4163

VOLUME 6

12. PROBABILITY – SPECIAL LECTURE

Elchanan Mossel, Combinatorial statistics and the sciences	4170
--	-------------

12. PROBABILITY

Jinho Baik, KPZ limit theorems	4190
Jian Ding, Julien Dubédat, Ewain Gwynne, Introduction to the Liouville quantum gravity metric	4212
Ronen Eldan, Analysis of high-dimensional distributions using pathwise methods	4246
Alison Etheridge, Natural selection in spatially structured populations	4272
Tadahisa Funaki, Hydrodynamic limit and stochastic PDEs related to interface motion	4302
Patrícia Gonçalves, On the universality from interacting particle systems	4326
Hubert Lacoin, Mixing time and cutoff for one-dimensional particle systems	4350
Dmitry Panchenko, Ultrametricity in spin glasses	4376
Kavita Ramanan, Interacting stochastic processes on sparse random graphs	4394
Daniel Remenik, Integrable fluctuations in the KPZ universality class	4426
Laurent Saloff-Coste, Heat kernel estimates on Harnack manifolds and beyond ...	4452

13. COMBINATORICS – SPECIAL LECTURE

Melanie Matchett Wood, Probability theory for random groups arising in number theory	4476
--	-------------

13. COMBINATORICS

Federico Ardila-Mantilla, The geometry of geometries: matroid theory, old and new	4510
Julia Böttcher, Graph and hypergraph packing	4542
Ehud Friedgut, KKL’s influence on me	4568
Allen Knutson, Schubert calculus and quiver varieties	4582

Sergey Norin, Recent progress towards Hadwiger’s conjecture	4606
Isabella Novik, Face numbers: the upper bound side of the story	4622
Mathias Schacht, Restricted problems in extremal combinatorics	4646
Alex Scott, Graphs of large chromatic number	4660
Asaf Shapira, Local-vs-global combinatorics	4682
Lauren K. Williams, The positive Grassmannian, the amplituhedron, and cluster algebras	4710

14. MATHEMATICS OF COMPUTER SCIENCE – SPECIAL LECTURES

Cynthia Dwork, Differential privacy: getting more for less	4740
Aayush Jain, Huijia Lin, Amit Sahai, Indistinguishability obfuscation	4762
David Silver, Andre Barreto, Simulation-based search control	4800
Bernd Sturmfels, Beyond linear algebra	4820

14. MATHEMATICS OF COMPUTER SCIENCE

Roy Gotlib, Tali Kaufman, Nowhere to go but high: a perspective on high-dimensional expanders	4842
Jelani Nelson, Forty years of frequent items	4872
Oded Regev, Some questions related to the reverse Minkowski theorem	4898
Muli (Shmuel) Safra, Mathematics of computation through the lens of linear equations and lattices	4914
Ola Svensson, Polyhedral techniques in combinatorial optimization: matchings and tours	4970
Thomas Vidick, $MIP^* = RE$: a negative resolution to Connes’ embedding problem and Tsirelson’s problem	4996
List of contributors	5027

VOLUME 7

15. NUMERICAL ANALYSIS AND SCIENTIFIC COMPUTING

Gang Bao, Mathematical analysis and numerical methods for inverse scattering problems	5034
---	-------------

Marsha J. Berger, Randall J. LeVeque, Towards adaptive simulations of dispersive tsunami propagation from an asteroid impact	5056
Jan S. Hesthaven, Cecilia Pagliantini, Nicolò Ripamonti, Structure-preserving model order reduction of Hamiltonian systems	5072
Nicholas J. Higham, Numerical stability of algorithms at extreme scale and low precisions	5098
Gitta Kutyniok, The mathematics of artificial intelligence	5118
Rachel Ward, Stochastic gradient descent: where optimization meets machine learning	5140
Lexing Ying, Solving inverse problems with deep learning	5154

16. CONTROL THEORY AND OPTIMIZATION – SPECIAL LECTURE

Nikhil Bansal, Discrepancy theory and related algorithms	5178
--	-------------

16. CONTROL THEORY AND OPTIMIZATION

Regina S. Burachik, Enlargements: a bridge between maximal monotonicity and convexity	5212
Martin Burger, Nonlinear eigenvalue problems for seminorms and applications ...	5234
Coralia Cartis, Nicholas I. M. Gould, Philippe L. Toint, The evaluation complexity of finding high-order minimizers of nonconvex optimization	5256
Yu-Hong Dai, An overview of nonlinear optimization	5290
Qi Lü, Control theory of stochastic distributed parameter systems: recent progress and open problems	5314
Asuman Ozdaglar, Muhammed O. Sayin, Kaiqing Zhang, Independent learning in stochastic games	5340
Marius Tucsnak, Reachable states for infinite-dimensional linear systems: old and new	5374

17. STATISTICS AND DATA ANALYSIS

Francis Bach, Lénaïc Chizat, Gradient descent on infinitely wide neural networks: global convergence and generalization	5398
Bin Dong, On mathematical modeling in image reconstruction and beyond	5420
Stefanie Jegelka, Theory of graph neural networks: representation and learning ...	5450
Oleg V. Lepski, Theory of adaptive estimation	5478

Gábor Lugosi, Mean estimation in high dimension	5500
Richard Nickl, Gabriel P. Paternain, On some information-theoretic aspects of non-linear statistical inverse problems	5516
Bernhard Schölkopf, Julius von Kügelgen, From statistical to causal learning	5540
Cun-Hui Zhang, Second- and higher-order Gaussian anticoncentration inequalities and error bounds in Slepian's comparison theorem	5594

18. STOCHASTIC AND DIFFERENTIAL MODELLING

Jacob Bedrossian, Alex Blumenthal, Sam Punshon-Smith, Lower bounds on the Lyapunov exponents of stochastic differential equations	5618
Nicolas Champagnat, Sylvie Méléard, Viet Chi Tran, Multiscale eco-evolutionary models: from individuals to populations	5656
Hyeonbae Kang, Quantitative analysis of field concentration in presence of closely located inclusions of high contrast	5680

19. MATHEMATICAL EDUCATION AND POPULARIZATION OF MATHEMATICS

Clara I. Grima, The hug of the scutoid	5702
Anna Sford, The long way from mathematics to mathematics education: how educational research may change one's vision of mathematics and of its learning and teaching	5716

20. HISTORY OF MATHEMATICS

June Barrow-Green, George Birkhoff's forgotten manuscript and his programme for dynamics	5748
Annette Imhausen, Some uses and associations of mathematics, as seen from a distant historical perspective	5772
Krishnamurthi Ramasubramanian, The history and historiography of the discovery of calculus in India	5784
List of contributors	5813