Preface

The origins of this book go back to a course on the history of mathematics that I gave at UCLA in the winter quarter of 2001. In most universities such a course follows a standard curriculum that starts with the Babylonians and works its way to more recent topics. I decided to do it differently and focus attention on the work of a single great figure. I chose Euler, attracted both by his universality and the great relevance of his work to what should be, if not what is, the undergraduate program in mathematics today. I discussed mostly Euler's work on infinite series and products, and was guided by the chapter on Euler in A. Weil's beautiful book *Number Theory : An Approach through History from Hammurapi to Legendre* that had been published in 1984. The result, as far as I was concerned, was predictable. I fell under the magical spell of Euler's personality and mathematics. It was a huge personal discovery for me to learn how alive Euler's themes still are. I started writing this book after the course, having been encouraged by my friends that it would be a good thing to try to do.

Before I get to Euler I really want to emphasize an important point. I believe that writing historically about mathematics should not be limited to questions like who did what when and to whom he/she wrote about it. I am of the opinion that the history of mathematics and mathematicians should go beyond these concerns, however legitimate they are. I feel that a complete mathematical history should pay great attention to the historical evolution of ideas and how they mesh with what we know and are interested in today. To modify a famous remark, the history of mathematics is too important to be left entirely to historians. No one has done this type of historical writing more brilliantly than A. Weil, as one can tell by his historical memoirs that are scattered throughout his *Oeuvres Scientifiques.* I have tried to follow his example; how far I have succeeded can be judged only by the readers.

No single person or book can describe completely the many-sided genius of Euler or his sunny and equable temperament that informs it. I have thus tried to limit my focus. I have been concerned only with the task of telling what the themes of Euler were and how they can be connected to current interests. Moreover, I have limited myself mostly to his work on infinite series and products and its repercussions in modern times, namely the theory of zeta values, and divergent series and integrals (Chapters 3 and 5). In Chapter 2 I have given a brief overview of some other parts of his work, for instance in elliptic integrals and number theory. His work on elliptic integrals is the forerunner of the modern theory of elliptic curves and abelian varieties, and his work on number theory raised questions which could only be understood fully after the development of class field theory. In Chapter 6 I have sketched a brief account of the theory of Euler products which he started but which really started to unfold with the work of Dirichlet and which, in the course of a long history, finally reached its climactic developments with the work of the number theorists of the late 19$^{\text{th}}$ century (class field theory) and the currently very active Langlands program. Parts of this chapter may be regarded as a very brief introduction to the Langlands program.

When writing any book, the question is always this: for whom is the book being written? To a large extent I was guided by a desire to reach the beginning graduate students and the advanced undergraduates, and to communicate to them the marvelous fact that things like class field theory, Borel summation, elliptic curves, and so on, did not come out fully grown from the primeval ocean (of the ancient Hindus for example) but grew out of small beginnings, and many of these go back to Euler. I feel that this historically motivated method of teaching is the best suited to convey the organic structure of mathematics. This of course is not the preferred way of teaching nowadays, where the students learn coherent sheaves and adeles before learning the Euler-Fermat theorem of primes which are sums of two squares. It actually happened in a high-powered course on D-modules I was attending a few years back that the first differential equation that was written, a few weeks after some very heavy stuff on D-modules, was Euler's, and the lecturer got it wrong because he forgot that the invariant operator on $\mathbf{C}^\times$ is not d/dz but $z(d/dz)$! I am convinced that the only way to produce young mathematicians who are not imprisoned by the small number of ideas they learn in a conventional graduate education is to emphasize the unity of mathematics from the beginning, and for this, the historical method, called the biogenetic method by Shafaraevitch, is the only possible one.

Thus this book is not a conventional historical essay on Euler—of these there are many wonderful examples—but rather a discussion of some of the Eulerian themes and how they fit into the modern perspective. More than anyone else, I know what the shortcomings of my attempt are. For example, although I have tried to keep the exposition as elementary as possible, here and there are places where this has been impossible to maintain, and I have had to assume familiarity on the part of the reader of more advanced material. But I have tried to organize everything in such a manner that a beginning graduate student, as well as a mathematician who does not always have a specialized knowledge of the topics treated, will find things understandable as well as enjoyable.

It only remains for me to give thanks to the people who helped me in this effort: to Don Babbitt and Sergei Gel'fand, who kept encouraging me throughout this enterprise; to Marina Eskin, who attended my course on Euler with an enthusiasm that was infectious; to Pierre Deligne, who was extraordinarily generous, as he always is, in sharing with me his insights on many aspects of Euler's work during his visit to UCLA in the spring of 2005; to Anita Colby, science librarian at UCLA, who always gave her help freely, instantly, and with a smile, for her help in bringing many items of Euleriana to my attention; to Richard Tsai for helping me with calculations using MATHEMATICA; to Aaron Pearl for help in photographing pages from Euler's papers; to Boyan Kostadinov, my student, for his help in translating articles by and about Euler from Russian as well as in proofreading; and to my wife, Veda, whose understanding and support have been the great steadying influences in my life.

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