Foreword

In the study of a mathematical system, algebraic structures allow for the discovery of more information. This is the motor behind the success of many areas of mathematics such as algebraic geometry, algebraic combinatorics, algebraic topology and others. This was certainly the motivation behind the observation of G.-C. Rota stating that various combinatorial objects possess natural product and coproduct structures. These structures give rise to a graded Hopf algebra, which is usually referred to as a combinatorial Hopf algebra. Typically, it is a graded vector space where the homogeneous components are spanned by finite sets of combinatorial objects of a given type and the algebraic structures are given by some constructions on those objects.

Recent foundational work has constructed many interesting combinatorial Hopf algebras and uncovered new connections between diverse subjects such as combinatorics, algebra, geometry, and theoretical physics. This has expanded the new and vibrant subject of combinatorial Hopf algebras. To give a few instances:

- Connes and Kreimer showed that a certain renormalization problem in quantum field theory can be encoded and solved using a Hopf algebra spanned by rooted trees.
- Loday and Ronco showed that a Hopf algebra based on planar binary trees is the free dendriform algebra on one generator. This is true for many types of algebras; the free algebra on one generator is a combinatorial Hopf algebra.
- In the context of polytope theory, some interesting enumerative combinatorial invariants induce a Hopf morphism from a Hopf algebra of posets to the Hopf algebra of quasi-symmetric functions.
- Krob and Thibon showed that the representation theory of the Hecke algebras at $q = 0$ is intimately related to the Hopf algebra structure of quasi-symmetric functions and non-commutative symmetric functions.

Some of the latest research in these areas has been the subject of a series of recent meetings, including an AMS/CMS meeting in Montréal in May 2002, a BIRS workshop in Banff in August 2004, and a CIRM workshop in Luminy in April 2005. It was suggested at the BIRS meeting that the draft text of M. Aguiar and S. Mahajan be expanded into the first monograph on the subject. Both are outstanding communicators. Their unified geometric approach using the combinatorics of Coxeter complexes and projection maps allows us to construct many of the combinatorial Hopf algebras currently under study and further to understand their properties (freeness, cofreeness, etc.) and to describe morphisms among them.

The current monograph is the result of this great effort and it is for me a great pleasure to introduce it.

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