Preface

Orthogonal, symplectic and unitary representations of finite groups combine two more traditional subjects of mathematics – linear representations of finite groups, and quadratic, skew symmetric and Hermitian forms – and thus inherit some of the characteristics of both. A little closer in approach to the latter, it could equally well be called the theory of quadratic, skew symmetric and Hermitian forms invariant under a (given) finite group.

This book is written as an introduction to the subject and not as an encyclopaedic reference text. The principal goal is an exposition of the known results on the equivalence theory, and related matters such as the Witt and Witt-Grothendieck groups, over the "classical" fields - algebraically closed, real closed, finite, local and global. I have included references to other results, especially those of a more "algebraic" nature.

I have chosen to ignore the integral theory entirely. This is unfortunate, especially from the point of view of surgery theory where the integral theory is essential ([71]) (and very well-developed – see [72]). But clearly either the size of this book or its level would have had to have been drastically changed to accommodate it.

Our exposition relies on the basic results in the linear representations of finite groups, in quadratic, symplectic and Hermitian forms and in involutions over simple algebras, which are outlined, mainly without proofs, in the first chapter. These subjects are blessed by many fine books on which we can rely for the background material – Curtis and Reiner's *Representation Theory of Finite Groups and Associative Algebras* [18] and their *Methods of Representation Theory with Applications to Finite Groups and Orders* [19], Reiner's *Maximal Orders* [55], Serre's *Linear Representations of Finite Groups* [68], Lam's *Introduction to Quadratic Forms over Fields* [39], O'Meara's *Introduction to Quadratic Forms* [54], Scharlau's *Quadratic and Hermitian Forms* [65], Knus's *Quadratic and Hermitian Forms over Rings* [35], and *The Book of Involutions* [36] of Knus, Merkurjev, Rost and Tignol.

It was A. Fröhlich who first gave a systematic organization of this subject, in a series of papers beginning in 1969. The paper *Orthogonal and symplectic representations of groups* [26] represents the culmination of his published work on orthogonal and symplectic representations. We have included most of the work from that paper, extending it to include unitary representations, and also providing new approaches, such as the use of the equivariant Brauer-Wall group in describing the principal invariants of orthogonal representations and their interplay with each other.

Chapter 1 is an outline, with few proofs, of the background material needed later. Most readers can skip it and refer to it later when necessary. The very brief

Chapter 2 introduces orthogonal, symplectic and unitary representations – which we refer to collectively as *isometry representations* – and gives the fundamental (and elementary) connection of these representations with Hermitian and skew Hermitian forms over the group algebra. Chapter 3 provides a firm foundation for the use of these latter forms in representation theory.

Chapter 4 begins the exposition of the theory proper, with a treatment of known results of the Witt-Grothendieck and Witt groups of isometry representations, followed by Chapter 5 in which the equivalence theory of isometry representations over finite, local and global fields is described, as well as a section on the isometry representations of the symmetric group. A final chapter 6 describes the equivariant Brauer-Wall group of equivariant central simple graded algebras and applies it to the description of the "classical invariants" of orthogonal representations and their interrelationships, through the medium of the equivariant Clifford algebra.

The main departure from the "usual" approach to the subject is the introduction, in Chapter 3, of a discriminant (or Gram) matrix for sesquilinear forms over simple algebras and its use in an explicit version of (Hermitian) Morita theory, which is critical in our treatment of this subject. In addition to the advantage of being completely elementary, it provides, in my opinion, more transparent proofs than does the more "invariant" and more general theory [28], and makes the detailed results of Chapter 5 possible.

I have tried to make this book as easy to read as possible by providing a list of notational conventions on p. ix and a glossary of symbols and a thorough index at the end.

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