Preface

It is easy to explain the concept of a configuration of points and lines to any ten-years-old youngster. Why then a book on this topic in a graduate series? There are several good reasons:

- First and foremost, configurations are mathematically challenging even though easily accessible.
- The study of configurations leans on many fields: classical geometry, combinatorics, topology, algebraic geometry, computing, and even analysis and number theory.
- There is a visual appeal to many types of configurations.
- There are opportunities for serious innovation that do not rely on long years of preliminary study.

The truly remarkable aspect of configurations is the scarcity of results in a field that was explicitly started well over a century ago, and informally much earlier. One of the foremost aims of the present text is to make available, essentially for the first time ever, a coherent account of the material.

Historical aspects are presented in order to enable the reader to follow the advances (as well as the occasional retreats) of the understanding of configurations. As explained more fully in the text, an initial burst of enthusiasm in the late nineteenth century produced several basic results. For almost a century, these were not matched by any comparably important new achievements. But near the end of the last century it turned out that the early results were incorrect, and this became part of the impetus for a reinvigorated study of configurations.

The recent realization that symmetries may play an important role in the investigations of configurations provided additional points of view on configurations. Together with the increased ability to actually draw configurations—made possible by advances in computer graphics—the stage was set for renewed efforts in correcting the ancient mistakes and to studying configurations that were never contemplated in the past.

This text relies very heavily on the graphical presentation of configurations. This is practically inevitable considering the topic and greatly simplifies the description of the many types of configurations dealt with. Most of the diagrams have been crafted using Mathematica®, Geometers Sketchpad®, and ClarisDraw®, often in combination.

In many respects this is a "natural history" of configurations—the properties and methods of generation depend to a large extent on the kind of configuration, and we present them in separate chapters and sections.

We have avoided insisting on proofs of properties that are visually obvious to such an extent that formal proofs would needlessly lengthen the exposition and make it quite boring. We firmly believe that an appropriate diagram is as much of a valid argument as a pedantic verbal explanation, besides being more readily understandable. It is hoped that the reader will agree!

The text is narrowly restricted to the topic of its title. There are many other kinds of configurations that might have been included. However, the nature of such configurations, for example, of points and planes, or of various higher-dimensional flats, is totally different from our topic. It is well possible that the early attempts to cover all possibilities led to very general definitions followed by very meager results.

Two exceptions to the restricted character of the presentation concern combinatorial configurations and topological configurations. The former are essential to the theory of geometric configurations, and we present the topic with this aim in mind. We do not enlarge on the various more general aspects of combinatorial designs and finite geometries—there are many excellent texts on these matters. Completely different is the situation regarding topological configurations. Very little is known about them, and the present text collects most of what is available.

One other aspect not covered here is the detailed investigations of the hierarchies of some special configurations. It seems that at one time it was fashionable to start with a simple result, such as the theorem of Pascal, and generate a whole family of objects by permuting the starting elements, then considering all the intersections of the resulting lines and the lines generated by the obtained points, etc. This way one could secure a family of points or

lines or whatever to be attached to one's name. The interested reader may gain access to this literature through other means.

For almost all the material covered, we provided as ample and detailed references as we were able to find. However, we did not give details concerning the programs that produced the various computer-generated enumerations. The reason—besides lack of competence—is that the programs and the computers on which they run change too rapidly for any printed information to be of lasting value. The interested reader should contact the authors of these results to obtain the most up-to-date status.

Results for which no reference is given are the author's and appear here for the first time. Also, as noted in appropriate places, several colleagues have been kind enough to allow the inclusion of their unpublished results—I am greatly indebted to them for this courtesy.

My gratitude goes to several other people and institutions. The American Mathematical Society was extremely helpful at all stages of the preparation of this text; in particular, allowing the illustrations to be in color has greatly increased the appeal of the book, as well as its instructional value. I greatly appreciate the attention of the editorial staff to detail and consistency, and the generous help they gave me through all stages of publication. The Department of Mathematics of the University of Washington supported my efforts in a variety of ways, both during the several times I gave graduate courses about configurations and in the preparation of the manuscript later on. The staff of the Mathematics Research Library at the university was very helpful in obtaining for me many of the old papers and books (and some new ones) and in guiding me through the labyrinths of the world of digital books and journals. Several stays at the Helen Riaboff Whiteley Center at the Friday Harbor Laboratories of the University of Washington provided the pleasant atmosphere and conditions conducive to work on this book.

Special thanks go to my friends and coauthors of recent papers on configurations—L. W. Berman, M. Boben, J. Bokowski, T. Pisanski, and L. Schewe. Their insights and comments, as well as results, encouraged me greatly while adding to the joint enterprises. The students of the courses I gave on configurations have earned my gratitude for their interest in the topic, which inspired me to investigate many questions and write up material in lecture notes.

Last—but certainly not least— my thanks go to my wife Zdenka, not only for her patience and forbearance over the long haul of my study of configurations and the preparation of the manuscript of this book, but even more for her love and support during well over half a century.

Branko Grünbaum

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