Contents

INTRODUCTION	ix
Chapter 0. A quick introduction to stratifolds	1
Chapter 1. Smooth manifolds revisited	5
§1. A word about structures	5
§2. Differential spaces	6
§3. Smooth manifolds revisited	8
$\S4.$ Exercises	11
Chapter 2. Stratifolds	15
§1. Stratifolds	15
$\S2.$ Local retractions	18
$\S3.$ Examples	19
§4. Properties of smooth maps	25
§5. Consequences of Sard's Theorem	27
§6. Exercises	29
Chapter 3. Stratifolds with boundary: c-stratifolds	33
§1. Exercises	38
Chapter 4. $\mathbb{Z}/2$ -homology	39
§1. Motivation of homology	39
§2. $\mathbb{Z}/2$ -oriented stratifolds	41
$\S3.$ Regular stratifolds	43
§4. $\mathbb{Z}/2$ -homology	45

$\S5.$	Exercises	51
Chapt	er 5. The Mayer-Vietoris sequence and homology groups of spheres	55
§1.	The Mayer-Vietoris sequence	55
$\S{2}$.	Reduced homology groups and homology groups of spheres	61
$\S3.$	Exercises	64
Chapt	er 6. Brouwer's fixed point theorem, separation, invariance of dimension	67
§1.	Brouwer's fixed point theorem	67
$\S{2}$.	A separation theorem	68
$\S3.$	Invariance of dimension	69
§4.	Exercises	70
Chapt	er 7. Homology of some important spaces and the Euler characteristic	71
§1.	The fundamental class	71
$\S{2}$.	$\mathbb{Z}/2$ -homology of projective spaces	72
$\S3.$	Betti numbers and the Euler characteristic	74
§4.	Exercises	77
Chapt	er 8. Integral homology and the mapping degree	79
§1.	Integral homology groups	79
$\S{2}$.	The degree	83
$\S3.$	Integral homology groups of projective spaces	86
§4.	A comparison between integral and $\mathbb{Z}/2$ -homology	88
$\S5.$	Exercises	89
Chapt	er 9. A comparison theorem for homology theories and	
	CW-complexes	93
§1.	The axioms of a homology theory	93
§2.	Comparison of homology theories	94
§3.	CW-complexes	98
§4.	Exercises	99
Chapt	Chapter 10. Künneth's theorem 10	
§1.	The cross product	103
§2.	The Künneth theorem	106
§3.	Exercises	109

Chapte	er 11. Some lens spaces and quaternionic generalizations	111
§1.	Lens spaces	111
§2.	Milnor's 7-dimensional manifolds	115
$\S3.$	Exercises	117
Chapte	er 12. Cohomology and Poincaré duality	119
§1.	Cohomology groups	119
§2.	Poincaré duality	121
$\S3.$	The Mayer-Vietoris sequence	123
§4.	Exercises	125
Chapte	er 13. Induced maps and the cohomology axioms	127
$\S1.$	Transversality for stratifolds	127
$\S{2}$.	The induced maps	129
$\S3.$	The cohomology axioms	132
§4.	Exercises	133
Chapte	er 14. Products in cohomology and the Kronecker pairing	135
$\S1.$	The cross product and the Künneth theorem	135
§2.	The cup product	137
$\S3.$	The Kronecker pairing	141
§4.	Exercises	145
Chapte	er 15. The signature	147
$\S1.$	Exercises	152
Chapte	er 16. The Euler class	153
§1.	The Euler class	153
§2.	Euler classes of some bundles	155
§3.	The top Stiefel-Whitney class	159
§4.	Exercises	159
Chapte	er 17. Chern classes and Stiefel-Whitney classes	161
$\S1.$	Exercises	165
Chapte	er 18. Pontrjagin classes and applications to bordism	167
§1.	Pontrjagin classes	167
$\S2.$	Pontrjagin numbers	170
§3.	Applications of Pontrjagin numbers to bordism	172
§4.	Classification of some Milnor manifolds	174

$\S5.$	Exercises	175
Chapte	er 19. Exotic 7-spheres	177
§1.	The signature theorem and exotic 7-spheres	177
§2.	The Milnor spheres are homeomorphic to the 7-sphere	181
§3.	Exercises	184
Chapte	er 20. Relation to ordinary singular (co)homology	185
§1.	$SH_k(X)$ is isomorphic to $H_k(X;\mathbb{Z})$ for CW -complexes	185
§2.	An example where $SH_k(X)$ and $H_k(X)$ are different	187
$\S3.$	$SH^k(M)$ is isomorphic to ordinary singular cohomology	188
§4.	Exercises	190
Appen	dix A. Constructions of stratifolds	191
§1.	The product of two stratifolds	191
§2.	Gluing along part of the boundary	192
$\S3.$	Proof of Proposition 4.1	194
Appen	dix B. The detailed proof of the Mayer-Vietoris sequence	197
Appen	dix C. The tensor product	209
Bibliog	graphy	215
Index		217