
Preface to the English edition

In addition to correcting some misprints and inaccuracies in the German edition, some parts of this book were revised and expanded. The sections dealing with gradient methods were shortened in order to make space for primal-dual active set strategies; the exposition of the latter now leads to the systems of linear equations to be solved. Following the suggestions of several readers, a derivation of the associated Green's functions is provided, using Fourier's method. Moreover, some references are discussed in greater detail, and some recent references on the numerical analysis of state-constrained problems have been added.

The sections marked with an asterisk may be skipped; their contents are not needed to understand the subsequent sections. Within the text, the reader will find formulas in framed boxes. Such formulas contain either results of special importance or the partial differential equations being studied in that section.

I am indebted to all readers who have pointed out misprints and supplied me with suggestions for improvements—in particular, Roland Herzog, Markus Müller, Hans Josef Pesch, Lothar v. Wolfersdorf, and Arnd Rösch. Thanks are also due to Uwe Prüfert for his assistance with the L^AT_EX typesetting. In the revision of the results on partial differential equations, I was supported by Eduardo Casas and Jens Griepentrog; I am very grateful for their cooperation. Special thanks are due to Jürgen Sprekels for his careful and competent translation of this textbook into English. His suggestions have left their mark in many places. Finally, I have to thank Mrs. Jutta Lohse for her careful proofreading of the English translation.

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F. Tröltzsch

Preface to the German edition

The mathematical optimization of processes governed by partial differential equations has seen considerable progress in the past decade. Ever faster computational facilities and newly developed numerical techniques have opened the door to important practical applications in fields such as fluid flow, microelectronics, crystal growth, vascular surgery, and cardiac medicine, to name just a few. As a consequence, the communities of numerical analysts and optimizers have taken a growing interest in applying their methods to optimal control problems involving partial differential equations; at the same time, the demand from students for this expertise has increased, and there is a growing need for textbooks that provide an introduction to the fundamental concepts of the corresponding mathematical theory.

There are a number of monographs devoted to various aspects of the optimal control of partial differential equations. In particular, the comprehensive text by J. L. Lions [**Lio71**] covers much of the theory of linear equations and convex cost functionals. However, the interest in the class notes of my lectures held at the technical universities in Chemnitz and Berlin revealed a clear demand for an introductory textbook that also includes aspects of nonlinear optimization in function spaces.

The present book is intended to meet this demand. We focus on basic concepts and notions such as:

- Existence theory for linear and semilinear partial differential equations
- Existence of optimal controls

- Necessary optimality conditions and adjoint equations
- Second-order sufficient optimality conditions
- Foundation of numerical methods

In this connection, we will always impose constraints on the control functions, and sometimes also on the state of the system under study. In order to keep the exposition to a reasonable length, we will not address further important subjects such as controllability, Riccati equations, discretization, error estimates, and Hamilton–Jacobi–Bellman theory.

The first part of the textbook deals with convex problems involving quadratic cost functionals and linear elliptic or parabolic equations. While these results are rather standard and have been treated comprehensively in [Lio71], they are well suited to facilitating the transition to problems involving semilinear equations. In order to make the theory more accessible to readers having only minor knowledge of these fields, some basic notions from functional analysis and the theory of linear elliptic and parabolic partial differential equations will also be provided.

The focus of the exposition is on nonconvex problems involving semilinear equations. Their treatment requires new techniques from analysis, optimization, and numerical analysis, which to a large extent can presently be found only in original papers. In particular, fundamental results due to E. Casas and J.-P. Raymond concerning the boundedness and continuity of solutions to semilinear equations will be needed.

This textbook is mainly devoted to the analysis of the problems, although numerical techniques will also be addressed. Numerical methods could easily fill another book. Our exposition is confined to brief introductions to the basic ideas, in order to give the reader an impression of how the theory can be realized numerically. Much attention will be paid to revealing hidden mathematical difficulties that, as experience shows, are likely to be overlooked.

The material covered in this textbook will not fit within a one-term course, so the lecturer will have to select certain parts. One possible strategy is to confine oneself to elliptic theory (linear-quadratic and nonlinear), while neglecting the chapters on parabolic equations. This would amount to concentrating on Sections 1.2–1.4, 2.3–2.10, and 2.12 for linear-quadratic theory, and on Sections 4.1–4.6 and 4.8–4.10 for nonlinear theory. The chapters devoted to elliptic problems do not require results from parabolic theory as a prerequisite.

Alternatively, one could select the linear-quadratic elliptic theory and add Sections 3.3–3.7 on linear-quadratic parabolic theory. Further topics

can also be covered, provided that the students have a sufficient working knowledge of functional analysis and partial differential equations.

The sections marked with an asterisk may be skipped; their contents are not needed to understand the subsequent sections. Within the text, the reader will find formulas in framed boxes. Such formulas contain either results of special importance or the partial differential equations being studied in that section.

During the process of writing this book, I received much support from many colleagues. M. Hinze, P. Maaß, and L. v. Wolfersdorf read various chapters, in parts jointly with their students. W. Alt helped me with the typographical aspects of the exposition, and the first impetus to writing this textbook came from T. Grund, who put my class notes into a first \LaTeX version. My colleagues C. Meyer, U. Prüfert, T. Slawig, and D. Wachsmuth in Berlin, and my students I. Neitzel and I. Yousept, proof-read the final version. I am indebted to all of them. I also thank Mrs. U. Schmickler-Hirzebruch and Mrs. P. Rußkamp of Vieweg-Verlag for their very constructive cooperation during the preparation and implementation of this book project.

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F. Tröltzsch