Preface

In February of 2007, I converted my "What's new" web page of research updates into a blog at `terrytao.wordpress.com`. This blog has since grown and evolved to cover a wide variety of mathematical topics, ranging from my own research updates, to lectures and guest posts by other mathematicians, to open problems, to class lecture notes, to expository articles at both basic and advanced levels.

With the encouragement of my blog readers, and also of the AMS, I published many of the mathematical articles from the first two years of the blog as [**Ta2008**] and [**Ta2009**], which will henceforth be referred to as *Structure and Randomness* and *Poincaré's Legacies Vols. I, II.* This gave me the opportunity to improve and update these articles to a publishable (and citeable) standard, and also to record some of the substantive feedback I had received on these articles by the readers of the blog.

The current text contains many (though not all) of the posts for the third year (2009) of the blog, focusing primarily on those posts of a mathematical nature which were not contributed primarily by other authors, and which are not published elsewhere. It has been split into two volumes.

The current volume consists of lecture notes from my graduate real analysis courses that I taught at UCLA (Chapter 1), together with some related material in Chapter 2. These notes cover the second part of the graduate real analysis sequence here, and therefore assume some familiarity with general measure theory (in particular, the construction of Lebesgue measure and the Lebesgue integral, and more generally the material reviewed in Section 1.1), as well as undergraduate real analysis (e.g., various notions of limits and convergence). The notes then cover more advanced topics in measure theory (notably, the Lebesgue-Radon-Nikodym and Riesz representation theorems) as well as a number of topics in functional analysis, such as the theory of Hilbert and Banach spaces, and the study of key function spaces such as the Lebesgue and Sobolev spaces, or spaces of distributions. The general theory of the Fourier transform is also discussed. In addition, a number of auxiliary (but optional) topics, such as Zorn's lemma, are discussed in Chapter 2. In my own course, I covered the material in Chapter 1 only and also used Folland's text [**Fo2000**] as a secondary source. But I hope that the current text may be useful in other graduate real analysis courses, particularly in conjunction with a secondary text (in particular, one that covers the prerequisite material on measure theory).

The second volume in this series (referred to henceforth as *Volume II*) consists of sundry articles on a variety of mathematical topics, which is only occasionally related to the above course, and can be read independently.

A remark on notation

For reasons of space, we will not be able to define every single mathematical term that we use in this book. If a term is italicised for reasons other than emphasis or for definition, then it denotes a standard mathematical object, result, or concept, which can be easily looked up in any number of references. (In the blog version of the book, many of these terms were linked to their Wikipedia pages, or other online reference pages.)

I will, however, mention a few notational conventions that I will use throughout. The cardinality of a finite set E will be denoted $|E|$. We will use the asymptotic notation $X = O(Y)$, $X \ll Y$, or $Y \gg X$ to denote the estimate $|X| \leq CY$ for some absolute constant $C > 0$. In some cases we will need this constant C to depend on a parameter (e.g., d), in which case we shall indicate this dependence by subscripts, e.g., $X = O_d(Y)$ or $X \ll_d Y$. We also sometimes use $X \sim Y$ as a synonym for $X \ll Y \ll X$.

In many situations there will be a large parameter n that goes off to infinity. When that occurs, we also use the notation $o_{n\to\infty}(X)$ or simply $o(X)$ to denote any quantity bounded in magnitude by $c(n)X$, where $c(n)$ is a function depending only on n that goes to zero as n goes to infinity. If we need $c(n)$ to depend on another parameter, e.g., d, we indicate this by further subscripts, e.g., $o_{n\to\infty;d}(X)$.

We will occasionally use the averaging notation $\mathbf{E}_{x\in X} f(x) := \frac{1}{|X|}\sum_{x\in X} f(x)$ to denote the average value of a function $f : X \to \mathbf{C}$ on a non-empty finite set X.

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