Contents

Preface		ix
Introduc	ction	1
Chapter	1. Semiflows on Metric Spaces	9
$\S{1.1.}$	Metric spaces	9
$\S{1.2.}$	Semiflows	17
$\S{1.3.}$	Invariant sets	19
$\S{1.4.}$	Exercises	25
Chapter	2. Compact Attractors	29
$\S{2.1.}$	Compact attractors of individual sets	30
$\S{2.2.}$	Compact attractors of classes of sets	36
$\S{2.3.}$	A sufficient condition for asymptotic smoothness	51
$\S{2.4.}$	α -limit sets of total trajectories	52
$\S{2.5.}$	Invariant sets identified through Lyapunov functions	52
$\S{2.6.}$	Discrete semiflows induced by weak contractions	54
$\S{2.7.}$	Exercises	57
Chapter	3. Uniform Weak Persistence	61
$\S{3.1.}$	Persistence definitions	61
$\S{3.2.}$	An SEIRS epidemic model in patchy host populations	64
§ 3 .3.	Nonlinear matrix models: Prolog	71
$\S{3.4.}$	The May-Leonard example of cyclic competition	78
$\S{3.5.}$	Exercises	84

Chapter 4	4. Uniform Persistence	87
§4.1.	From uniform weak to uniform persistence	87
$\S4.2.$	From uniform weak to uniform persistence: Discrete case	91
$\S4.3.$	Application to a metered endemic model of SIR type	94
$\S4.4.$	From uniform weak to uniform persistence for time-set \mathbb{R}_+	97
$\S 4.5.$	Persistence à la Baron von Münchhausen	99
$\S4.6.$	Navigating between alternative persistence functions	107
$\S4.7.$	A fertility reducing endemic with two stages of infection	110
$\S4.8.$	Exercises	123
Chapter &	5. The Interplay of Attractors, Repellers, and Persistence	125
$\S{5.1.}$	An attractor of points facilitates persistence	125
$\S{5.2.}$	Partition of the global attractor under uniform persistence	127
$\S{5.3.}$	Repellers and dual attractors	135
$\S{5.4.}$	The cyclic competition model of May and Leonard revisited	139
$\S{5.5.}$	Attractors at the brink of extinction	140
$\S{5.6.}$	An attractor under two persistence functions	141
$\S{5.7.}$	Persistence of bacteria and phages in a chemostat	142
$\S{5.8.}$	Exercises	155
Chapter	6. Existence of Nontrivial Fixed Points via Persistence	157
$\S6.1.$	Nontrivial fixed points in the global compact attractor	158
$\S6.2.$	Periodic solutions of the Lotka-Volterra predator-prey model	160
$\S6.3.$	Exercises	162
Chapter '	7. Nonlinear Matrix Models: Main Act	163
§7.1.	Forward invariant balls and compact attractors of bounded	
	sets	163
§7.2.	Existence of nontrivial fixed points	165
§7.3.	Uniform persistence and persistence attractors	167
§7.4.	Stage persistence	171
§7.5.	Exercises	175
Chapter a	8. Topological Approaches to Persistence	177
$\S{8.1.}$	Attractors and repellers	177
§8.2.	Chain transitivity and the Butler-McGehee lemma	180
§8.3.	Acyclicity implies uniform weak persistence	185
§8.4.	Uniform persistence in a food chain	191

§8.5.	The metered endemic model revisited	196
§8.6.	Nonlinear matrix models (epilog): Biennials	199
§8.7.	An endemic with vaccination and temporary immunity	209
§8.8.	Lyapunov exponents and persistence for ODEs and maps	215
§8.9.	Exercises	229
Chapter	9. An SI Endemic Model with Variable Infectivity	231
$\S{9.1.}$	The model	231
$\S{9.2.}$	Host persistence and disease extinction	236
$\S{9.3.}$	Uniform weak disease persistence	237
$\S{9.4.}$	The semiflow	239
$\S{9.5.}$	Existence of a global compact attractor	240
$\S{9.6.}$	Uniform disease persistence	245
$\S{9.7}.$	Disease extinction and the disease-free equilibrium	247
$\S{9.8.}$	The endemic equilibrium	249
$\S{9.9.}$	Persistence as a crossroad to global stability	250
$\S{9.10}.$	Measure-valued distributions of infection-age	254
Chapter	10. Semiflows Induced by Semilinear Cauchy Problems	261
$\S{10.1}.$	Classical, integral, and mild solutions	261
$\S{10.2}.$	Semiflow via Lipschitz condition and contraction principle	265
$\S{10.3.}$	Compactness all the way	266
$\S{10.4.}$	Total trajectories	271
$\S{10.5}.$	Positive solutions: The low road	273
$\S{10.6}.$	Heterogeneous time-autonomous boundary conditions	279
Chapter	11. Microbial Growth in a Tubular Bioreactor	283
$\S{11.1.}$	Model description	283
$\S{11.2.}$	The no-bacteria invariant set	287
$\S{11.3.}$	The solution semiflow	291
$\S{11.4.}$	Bounds on solutions and the global attractor	292
$\S{11.5.}$	Stability of the washout equilibrium	296
$\S{11.6.}$	Persistence of the microbial population	301
$\S{11.7.}$	Exercises	304
Chapter	12. Dividing Cells in a Chemostat	307
$\S{12.1.}$	An integral equation	309
$\S{12.2.}$	A C_0 -semigroup	314

$\S{12.3.}$	A semilinear Cauchy problem	318
$\S{12.4.}$	Extinction and weak persistence via Laplace transform	320
$\S{12.5.}$	Exercises	325
Chapter	13. Persistence for Nonautonomous Dynamical Systems	327
$\S{13.1.}$	The simple chemostat with time-dependent washout rate	327
$\S{13.2.}$	General time-heterogeneity	332
$\S{13.3.}$	Periodic and asymptotically periodic semiflows	335
$\S{13.4.}$	Uniform persistence of the cell population	336
$\S{13.5.}$	Exercises	339
Chapter	14. Forced Persistence in Linear Cauchy Problems	341
§14.1.	Uniform weak persistence and asymptotic Abel-averages	342
§14.2.	A compact attracting set	343
$\S{14.3.}$	Uniform persistence in ordered Banach space	344
Chapter	15. Persistence via Average Lyapunov Functions	349
$\S{15.1.}$	Weak average Lyapunov functions	350
$\S{15.2.}$	Strong average Lyapunov functions	354
$\S{15.3.}$	The time-heterogeneous hypercycle equation	355
$\S{15.4.}$	Exercises	361
Appendiz	x A. Tools from Analysis and Differential Equations	363
§A.1.	Lower one-sided derivatives	363
§A.2.	Absolutely continuous functions	364
§A.3.	The method of fluctuation	365
§A.4.	Differential inequalities and positivity of solutions	367
§A.5.	Perron-Frobenius theory	372
§A.6.	Exercises	375
Appendiz	x B. Tools from Functional Analysis and Integral Equations	377
§B.1.	Compact sets in $L^p(\mathbb{R}_+)$	377
§B.2.	Volterra integral equations	378
§B.3.	Fourier transform methods for integro-differential equations	380
§B.4.	Closed linear operators	385
§B.5.	Exercises	390
Bibliogra	phy	391
Index		403