Preface

This book is based on the lecture notes written for the advanced Ph.D. level statistics courses delivered by the first author at the Wayne State University over the last decade. It has been easy to observe how the gap deepens between applied (computational) and theoretical statistics. It has become more difficult to direct and mentor graduate students in the field of mathematical statistics. The research monographs in this field are extremely difficult to use as textbooks. Even in the best published lecture notes the intensive material of original studies is typically included. On the other hand, the classical courses in statistics that cover the traditional parametric point and interval estimation methods and hypotheses testing are hardly sufficient for the teaching goals in modern mathematical statistics.

In this book, we tried to give a general overview of the key statistical topics, parametric and nonparametric, as a set of very special optimization problems. As a criterion for optimality of estimators we chose minimax risks, and we focused on asymptotically minimax rates of convergence for large samples. Definitely, the selection of models presented in this book follows our preferences. Many very important problems and examples are not included. The simplest models were deliberately selected for presentation, and we consciously concentrated on the detailed proofs of all propositions. We believe that mathematics students should be trained in proof-writing to be better prepared for applications in statistics.

This textbook can form a reasonable basis for a two-semester course in mathematical statistics. Every chapter is followed by a collection of exercises consisting partly of verification of technical results, and partly of important

illustrative examples. In our opinion, the sufficient prerequisite is a standard course in advanced probability supported by undergraduate statistics and real analysis. We hope that students who successfully pass this course are prepared for reading original papers and monographs in the minimax estimation theory and can be easily introduced to research studies in this field.

This book is organized into three parts. Part 1 is comprised of Chapters 1-7 that contain fundamental topics of local asymptotic normality as well as irregular statistical models, change-point problem, and sequential estimation. For convenience of reference we also included a chapter on classical parametric linear regression with the concentration on the asymptotical properties of least-squares estimators. Part 2 (Chapters 8-12) focuses on estimation of nonparametric regression functions. We restrict the presentation to estimation at a point and in the quadratic and uniform norms, and consider deterministic as well as random designs. The last part of the book, Chapters 13-16, is devoted to special more modern topics such as influence of higher-dimension and structure in nonparametric regression models, problems of adaptive estimation, and testing of nonparametric hypotheses. We present the ideas through simple examples with the equidistant design.

Most chapters are weakly related to each other and may be covered in any order. Our suggestion for a two-semester course would be to cover the parametric part during the first semester and to cover the nonparametric part and selected topics in the second half of the course.

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