
Index

- adjoint, 58
- adjoint action, 174
- almost analytic extension, 34, 358, 359
- annihilation, creation operators, 120
- asymptotic sum, 73
- atlas, 340
- average
 - in time, 109, 366
 - of symbols, 372

- Beals's Theorem, 7, 171–177, 179, 180, 196, 216, 249, 271
- Bergman kernel, 293, 307–309
- Bergman projection, 307
- Bergman projector, 293
- Birkhoff's Ergodic Theorem, 367
- Borel's Theorem, 74, 231, 278, 284

- Cartan's formula, 22, 23, 255, 395, 397
- Cauchy–Riemann operator, 34, 289, 294
- characteristic
 - equations, 274
 - variety, 104
- closable operator, 405
- closed operator, 405
- Coarea Formula, 366
- coherent state, 102
- commutator, 5, 61, 68, 106, 174
- compact operator, 87–90, 129, 186, 403–405, 409, 411, 416
- composition formula, 6, 66, 160, 175, 178, 186, 193, 194, 263, 268
- conjugation, 141, 277, 282

- and symbols, 141
- by Fourier transform, 56, 64
- by unitary operators, 247, 251, 256, 257, 260, 266, 368
- contraction of forms, 20, 393
- coordinate patch, 340
- cotangent bundle, 341
 - canonical symplectic form, 342
 - integral over, 342
- cotangent space, 342
- Cotlar–Stein Theorem, 86, 90, 401

- defect measure, semiclassical, 99–117
 - definition, 101
 - examples, 102–104
 - on torus, 108
 - properties, 104–106
- density of states, 132
- diffeomorphism, 14, 18, 22, 24, 48, 192, 194, 197, 199, 203, 207, 340, 383, 386
- differential, 392
- differential forms, 15, 22, 24, 230, 342, 391–398
 - canonical, 342
- differential operator, 345
- distribution, 35, 58, 76, 345, 347, 387, 399
- domain of operator, 405
- Duhamel's formula, 154, 234, 270, 271
- dynamics
 - classical, 1, 4, 8, 13–14, 106, 272, 274
 - operator, 181, 220–227, 246

- quantum, 5, 106, 219–226, 245–251, 264–271, 409
- Egorov’s Theorem, 8, 245–251, 278
 for long times, 8, 264–271
 weak, 368–370
- Ehrenfest time, 8, 264–271
- eigenfunctions, 7, 119–137, 143, 152
 basis of, 129, 157, 357
 clusters of, 157, 243
 concentration in phase space, 125
 equidistribution of, 365, 378, 379
 exponential decay estimates, 143
 for harmonic oscillator, 120–124, 127
 for Laplace–Beltrami operator, 357, 378
 for pseudodifferential operator, 187
 on manifolds, 356
 order of vanishing, 152
 regularity, 354
- eigenvalues, 48, 120, 123, 286, 287, 357, 362, 363, 370, 404, 405, 411, 412
 and trace, 413
 counting, 410
 for harmonic oscillator, 120–124
 for Laplace–Beltrami operator, 157, 243, 357, 362
 for Schrödinger’s equation, 1, 7, 119, 286, 357
 minimax formulas for, 409, 410
 of matrix, 36
 of operator, 403
- elliptic
 estimates, 140
 symbol, 91, 133, 144, 146, 156, 222, 242, 260, 277, 282, 288, 289
- energy
 decay, 114–117
 surface, 366
 wave equation, 109
- ergodicity, 8
 classical, 366–368
 quantum, 365–379
- essential support, 192–194
- essentially selfadjoint operator, 406
- estimates
 H_h^2 , 140
 H_h^k , 149
 Agmon–Lithner, 142
 Carleman, 7, 146–148
 notation for, 388
 Schauder, 7, 158–167
 Strichartz, 8, 235–240
- exponential map, 14
- FBI transform, 291, 302–311
- flow map, 14
- forbidden region, 139, 141
- Fourier
 decomposition, 65
 integral operator, 8, 199, 228, 244, 245
- Fourier transform, 2, 27–40
 exponential of imaginary quadratic form, 36
 exponential of real quadratic form, 28
 on \mathcal{S} , 28
 on \mathcal{S}' , 36
 semiclassical, 38–40
- Fredholm
 operator, 416
 theory, 415–419
- functional calculus, 137, 354, 357–361, 370
- generalized Sobolev space $H_h(m)$, 182–187
 definition, 183
 dual space of, 184
 examples, 183
 pseudodifferential operators and, 185, 187
- geodesic flow, 365, 378, 379
- graph, 405
 twisted, 25, 262, 383
- Grushin problems, 415–417
- half-density, 197–206, 234–235, 345
- Hamilton–Jacobi equation, 7, 228, 231–233, 238, 260
- harmonic oscillator, 119–124, 126, 135
 Weyl’s Law for, 123
- heat equation, 285
- Helffer–Sjöstrand formula, 358, 361, 363
- Helmholtz’s equation, 274
- Hermite polynomials, 121
- hypoellipticity
 condition, 144, 145
 estimate, 144
- Implicit Function Theorem, 19, 25, 156, 366
- index of Fredholm operator, 416
- inequality
 Fefferman–Phong, 93, 216

- Gårding, 6, 73, 92–96, 101, 142, 145, 214–216
- Gronwall, 117, 265
- Hardy–Littlewood–Sobolev, 236
- Minkowski, 239
- Schur, 82, 314, 327
- interpolation, 236, 241, 244
- inverse, 91–92
 - approximate, 400
- Inverse Function Theorem, 49, 400, 420
- Jacobi’s identity, 20, 21
- kernel
 - Bergman, 293, 307–309
 - of Fredholm operator, 416
 - Schwartz, 59, 65, 81, 82, 175, 209, 210, 238, 263, 370, 399
- Laplace–Beltrami operator, 157, 243, 351, 353, 357, 362, 378
- Leibnitz rule, 150
- Lidskii’s Theorem, 413
- Lie derivative, 395
- lifting, 18–19, 203
- Liouville measure, 366
- Littlewood–Paley theory, 158, 159, 161, 163, 167, 214
- localization, 39, 153, 155, 188, 195
- manifolds, 339–363
 - definition of, 339
 - PDE on, 353–362
 - pseudodifferential operators on, 345–352
 - Riemannian, 344–345
 - smooth functions on, 340
- Maslov index, 264
- matrices
 - J , 15, 16, 64, 257
 - notation for, 384
 - symplectic, 252–253, 262
 - transition, 340
- Mean Ergodic Theorem, 367
- meromorphic
 - family of operators, 110, 419
 - resolvents, 129, 131
- microlocality, 195
- microlocally invertible, 195
- Morse Lemma, 46, 48–50
- nondegeneracy condition, 15, 48, 155, 157, 237, 239, 240, 244, 282
- nonnormal operators, 287
- norm, 140, 346, 347, 371, 387, 411
- normal forms, 273–289
 - complex symbols, 282–286
 - real symbols, 275–279
- notation, 383–389
 - basic, 383–384
 - for estimates, 388
 - for functions, 385–387
 - for matrices, 384
 - for operators, 387
 - for sets, 384
 - multiindex, 385
- observables, 3, 5, 8, 56, 247, 348, 370
- Open Mapping Theorem, 417
- order functions, 73
 - change of, 182, 183
 - definition, 72
 - examples, 72
 - log of, 182
- order of vanishing, 148–152
- oscillatory integral, 6, 40, 46, 52–54
- oscillatory testing, 80
- phase shift, 36
- Planck’s constant, 1, 5
- plurisubharmonic functions, 300–302
- Poincaré’s Lemma, 230, 255, 396
 - on manifolds, 398
- Poisson bracket, 4, 5, 20, 68, 106, 369, 386
- polar decomposition, 252
- principal symbol, 74, 213, 277, 279, 281, 282, 361, 371
- principal type, 276, 278, 282
- projection, 127–129, 131, 134, 368, 371, 373
 - Bergman, 293, 307
- propagation of singularities, 279–281
- pseudodifferential operators, 2, 4, 55–96
 - on manifold, 347
 - symbol of, 348, 349, 351
- pseudolocality, 81, 204
- pseudospectrum, 287, 288
- push-forward, 246, 394
- quadratic forms, 295
- quantization
 - and commutators, 61
 - composition, 66
 - Fourier decomposition, 65

- general, 56
- linear symbols, 59, 60
- on torus, 106–108
- standard, 56
- symbols
 - exponentials of linear symbols, 62
 - exponentials of quadratic symbols, 63
- symbols depending on x only, 59
- symbols linear in x , 60
- Toeplitz, 8, 293, 311–320
- Weyl, 4, 6, 56
 - complex, 312–316
- quantum mechanics, 1, 198
 - Heisenberg picture, 5, 247, 271, 368
- quasimode, 152–157, 240–243, 286–288
- Quillen’s Theorem, 332
- rank, 410
- Rank-Nullity Theorem, 418
- Rauch–Taylor Theorem, 354
- rescaling, 2, 38, 39, 57, 95, 123, 126
 - standard, 57
- Riemannian manifold, 152, 157, 243, 344–345, 365, 378
- Riesz Representation Theorem, 101
- Riesz–Thorin Theorem, 236
- s -density bundles, 342
- Schrödinger’s equation, 1, 7
- Schur complement formula, 415
- Schwartz space \mathcal{S} , 28
- section, 341
- selfadjoint operator, 58, 106, 130, 177, 221, 222, 244, 286, 368, 401, 402, 404–409, 411–413
- seminorm, 28, 76, 108, 131, 192, 211, 388
- signature of matrix, 36
- singular values, 411
- Sobolev space, 140, 183, 346, 351, 355
 - generalized, 7, 182–187, 279
- Sogge’s Theorem, 243
- spectral clusters, 157, 243
- spectrum, 129–132, 177, 286, 287, 357, 403, 405, 408–411
- stationary phase, 2, 6, 40–52, 68, 69, 72, 78, 103, 213, 239
 - higher-dimensional, 46–52
 - one-dimensional, 40–46
- Stirling’s formula, 150, 151, 328
- Stone’s Theorem, 222, 409
- subadditive function, 265
- symbol calculus, 55
- symbols, 3, 56, 389
 - depending only on x , 59
 - distributional, 58
 - exponentials of linear symbols, 62
 - exponentials of quadratic symbols, 63
 - Kohn–Nirenberg, 7, 206–217, 389
 - linear, 59, 60
 - linear in x , 60
- symmetric operator, 132, 222, 355, 406
- symplectic
 - form, 342
 - geometry, 2, 13–26
 - complex, 299
 - mapping, 16–20
 - matrix, 16–17, 252, 262
 - product σ , 14, 50
- tangent
 - bundle, 341
 - space, 341
- Taylor’s Theorem, 93, 95, 151
- tempered
 - distributions, 35
 - family of distributions, 187
 - family of operators, 187, 188
- Toeplitz quantization, 293, 311–320
- torus, 7, 106–109, 366, 383
- trace, 413
 - integral operators, 413
- trace class, 361, 411–413
 - norm, 411
- transform
 - Bargmann, 292, 306
 - FBI (Fourier–Bros–Iagolnitzer), 291, 302–311
 - Fourier, 2
 - Gabor, 292
 - Segal–Bargmann, 292
- tunneling, 2, 7, 143–148
- uncertainty principle, 39–40, 132, 196
- unitary
 - matrix, 253
 - operators, 85, 100, 126, 172, 176, 220, 222, 228, 246, 251, 256, 259, 260, 263, 368, 404, 408, 409
- vector bundles, 340–343
 - fibers of, 340
 - sections of, 341

- transition matrices, 340
- version, 162

- wave equation, 281
 - damped, 2, 7, 109–117
- wavefront set
 - classical, 190
 - for operators, 194
 - semiclassical, 188, 191, 192, 196
 - using FBI transform, 323
- wedge product, 391
- weight, 145
- Weyl's Law, 7, 132–137, 370
 - for harmonic oscillator, 123
 - on manifolds, 361–362
- WKB approximation, xii, 227, 228, 273–274

- Young's inequality, 160