Preface

In the long-forgotten days of pre-history, people would color peach pits differently on the two sides, toss them in the air, and bet on the color that came up. We, with a more advanced technology, toss coins. We flip a coin into the air. There are only two possible outcomes, heads or tails, but until the coin falls, we have no way of knowing which. The result of the flip may decide a bet, it may decide which football team kicks off, which tennis player serves, who does the dishes, or it may decide a hero's fate.

The coin flip may be the most basic of all random experiments. If the coin is reasonably well-made, heads is as likely as tails to occur. But... what does that mean?

Suppose we flip a coin, and call "Heads" or "Tails" while it is in the air. Coins are subject to the laws of physics. If we could measure the exact position, velocity, and angular velocity of the coin as it left the hand—its initial conditions—we could use Newton's laws to predict exactly how it would land. Of course, that measurement is impractical, but not impossible. The point is that the result is actually determined as soon as the coin is in the air and, in particular, it is already determined when we call it; the result is (theoretically) known, but not to us. As far as we are concerned, it is just as unpredictable as it was before the flip. Let us look at the physics to see why.

The outcome is determined by the exact position, angular position, velocity, and angular velocity at the time of the flip. Physicists represent these all together as a point in what they call phase space. We can picture it as follows.

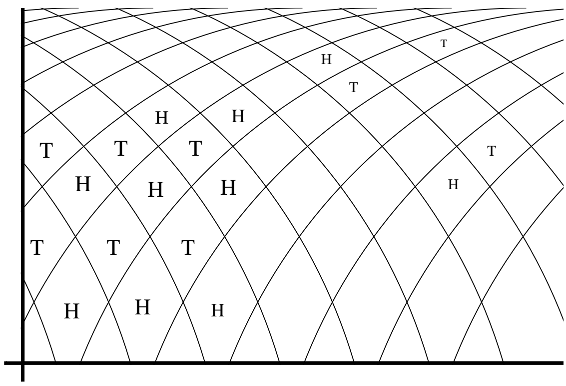

Figure 1. Phase space

This represents the initial condition of the coin in phase space. Some points lead to heads, some to tails. But a small difference in initial conditions completely changes the result. The conditions leading to heads are a union of very small regions, which are evenly mixed up with those leading to tails.

This means that no matter how we try to toss the coin, we cannot zero in on a particular result—our toss will be smeared out, so to speak, over the "Heads" and "Tails" regions, and this will happen no matter how carefully we toss it. This leads us to say things like: "Heads and tails are equally likely," or "Heads and tails each have probability one-half."

Philosophers ask deep questions about the meaning of randomness and probability. Is randomness something fundamental? Or is it just a measure of our ignorance? Gamblers just want to know the odds.

Mathematicians by and large prefer to duck the question. If pressed, they will admit that most probability deals with chaotic situations, like the flip of a coin, where the seeming randomness comes from our ignorance of the true situation. But they will then tell you that the *really* important thing about randomness is that it can be measured—for probabilities measure likelihood—and that we can construct a mathematical model which enables us to compute all of the probabilities, and that, finally, this model is the proper subject of study.

So you see, mathematicians side with the gamblers: they just want to know the odds.

From now on, probability is mathematics. We will be content just to note that it works—which is why so few casino owners go broke—and we will leave the deeper meanings of randomness to the philosophers.