Preface

About

When you publish a textbook on such a classical subject the first question you will be faced with is: Why another book on this subject? Everything started when I was supposed to give the basic course on *Ordinary Differential Equations* in Summer 2000. (At that time the course met 5 hours per week.) While there were many good books on the subject available, none of them quite fit my needs. I wanted a concise but rigorous introduction with full proofs that also covered classical topics such as Sturm–Liouville boundary value problems, differential equations in the complex domain, as well as modern aspects of the qualitative theory of differential equations. The course was continued with a second part on *Dynamical Systems and Chaos* in Winter 2000/01, and the notes were extended accordingly. Since then the manuscript has been rewritten and improved several times according to the feedback I got from students over the years when I redid the course. Moreover, since I had the notes on my homepage from the very beginning, this triggered a significant amount of feedback as well, from students who reported typos, incorrectly phrased exercises, etc., to colleagues who reported errors in proofs and made suggestions for improvements, to editors who approached me about publishing the notes. All this interest eventually resulted in a Chinese translation of an earlier version of the book. Moreover, if you google for the manuscript, you can see that it is used at several places worldwide, linked as a reference at various sites, including Wikipedia. Finally, Google Scholar will tell you that it is even cited in several publications. Hence I decided that it was time to turn it into a *real* book.

Content

This book's main aim is to give a self-contained introduction to the field of ordinary differential equations with emphasis on the dynamical systems point of view while still keeping an eye on classical tools as pointed out before.

The first part is what I typically cover in the introductory course for bachelor's level students. Of course it is typically not possible to cover everything and one has to skip some of the more advanced sections. Moreover, it might also be necessary to add some material from the first chapter of the second part to meet curricular requirements.

The second part is a natural continuation beginning with planar examples (culminating in the generalized Poincaré–Bendixson theorem), continuing with the fact that things get much more complicated in three and more dimensions, and ending with the stable manifold and the Hartman–Grobman theorem.

The third and last part gives a brief introduction to chaos, focusing on two selected topics: Interval maps with the logistic map as the prime example plus the identification of homoclinic orbits as a source for chaos and the Melnikov method for perturbations of periodic orbits and for finding homoclinic orbits.

Prerequisites

This book requires only some basic knowledge of calculus, complex functions, and linear algebra. In addition, I have tried to show how a computer system, *Mathematica*[1], can help with the investigation of differential equations. However, the course is not tied to *Mathematica* and any similar program can be used as well.

Updates

The AMS is hosting a Web page for this book at

`http://www.ams.org/bookpages/gsm-140/`

where updates, corrections, and other material may be found, including a link to material on my website:

`http://www.mat.univie.ac.at/~gerald/ftp/book-ode/`

[1] *Mathematica*® is a registered trademark of Wolfram Research, Inc.

There you can also find an accompanying *Mathematica* notebook with the code from the text plus some additional material.

Acknowledgments

I wish to thank my students, Ada Akerman, Kerstin Ammann, Jörg Arnberger, Alexander Beigl, Paolo Capka, Jonathan Eckhardt, Michael Fischer, Anna Geyer, Ahmed Ghneim, Hannes Grimm-Strele, Tony Johansson, Klaus Kröncke, Alice Lakits, Simone Lederer, Oliver Leingang, Johanna Michor, Thomas Moser, Markus Müller, Andreas Németh, Andreas Pichler, Tobias Preinerstorfer, Jin Qian, Dominik Rasipanov, Martin Ringbauer, Reinaldo Garcia Rosario, Simon Rößler, Robert Stadler, Shelby Stanhope, Raphael Stuhlmeier, Gudrun Szewieczek, Gerhard Tulzer, Paul Wedrich, Florian Wisser, and colleagues, Edward Dunne, Klemens Fellner, Giuseppe Ferrero, Ilse Fischer, Delbert Franz, Heinz Hanßmann, Daniel Lenz, Jim Sochacki, and Eric Wahlén, who have pointed out several typos and made useful suggestions for improvements. Finally, I would like to thank the anonymous referees for valuable suggestions that improved the presentation of the material.

If you find any errors or if you have comments or suggestions (no matter how small), please let me know.

I have been supported by the Austrian Science Fund (FWF) during much of this writing, most recently under Grant Y330.

Gerald Teschl

Vienna, Austria
April 2012

Gerald Teschl
Fakultät für Mathematik
Nordbergstraße 15
Universität Wien
1090 Wien, Austria

E-mail: `Gerald.Teschl@univie.ac.at`
URL: `http://www.mat.univie.ac.at/~gerald/`