

---

# Index

- adjoint operator, 66, 69, 198
- advection, 186
- algebra of functions, 47
- approximation, 48
  - backward Euler, 117, 125, 128, 135
  - by polynomials, 51
  - by trigonometric polynomials, 52, 58
  - forward Euler, 117
  - of Sobolev functions, 157
  - with smooth functions, 157
- Ascoli, 54
  
- Baire, 62, 221
- ball
  - closed, 13
  - open, 13
- Banach space, 13
- Banach-Alaoglu, 34
- Banach-Steinhaus, 61
- basis
  - orthonormal, 85, 88
- Bessel, 87
- biharmonic equation, 216
- Bochner, 227
- boundary condition, 189
  
- Cantor, 141
- Cartesian product, 36
- Cauchy, 78, 234
- Cauchy problem, 115
- closed graph theorem, 64
- closure, 218
- contraction mapping theorem, 115, 219
  
- convergence
  - of weak derivatives, 146
  - pointwise, 46
  - strong, 33
  - uniform, 46
  - weak, 33, 92
  - weak star, 33
- convex hull, 37
  
- derivative
  - distributional, 140
  - of a distribution, 144
  - pointwise, 140
  - strong, 159
  - weak, 140, 144, 156, 159
- diffusion, 186
- Dini, 46
- Dirac, 141
- Dirichlet's boundary condition, 186
- distance, 11, 217
  - induced by seminorms, 24
- distribution, 143
  - order of, 143
- domain of an operator, 16
  
- Egoroff, 223
- eigenfunction, 192
- eigenvalue, 106
- eigenvector, 106
- elliptic equation, 185
- embedding, 163
  - compact, 175, 178
  - Gagliardo-Nirenberg, 172

- Morrey, 168
- Sobolev, 172
- energy, 209
- epigraph, 233
- equicontinuity, 53
- essential spectrum, 106
- essential supremum, 223
- Euler, 117
- exponential
  - of a linear operator, 118
  - of a matrix, 118
- extension
  - of a linear functional, 27
  - of a Sobolev function, 161
- extreme point, 97
  
- Fatou, 224
- fixed point, 219
- Fourier, 88
- Fréchet, 25
- Fredholm, 101
  - alternative, 105, 196
- Fubini, 226
- function
  - absolutely continuous, 148, 225
  - almost separably valued, 227
  - Banach space valued, 227
  - Cantor, 141
  - constant, 147
  - continuous, 219
  - Heaviside, 140
  - Hölder continuous, 56, 219
  - integrable, 224
  - Lipschitz continuous, 115, 181, 219
  - locally summable, 139, 142, 224
  - measurable, 222
  - simple, 223, 227
  - strongly measurable, 227
  - summable, 224
  - test, 142
  - uniformly convex, 233
  - weakly differentiable, 149
  - weakly measurable, 227
  
- Gagliardo-Nirenberg, 163
- generator of a semigroup, 122
- Gram determinant, 98
- Gram-Schmidt orthogonalization, 85
- graph, 64
- Gronwall, 237
  
- Hahn-Banach, 27
  
- Hausdorff Maximality Principle, 28, 217
- Heaviside, 140
- Hilbert, 79, 109
- Hölder, 56, 235
- hyperbolic equation, 207
- hyperplane
  - supporting, 234
  
- inequality
  - Bessel, 87
  - Cauchy, 234
  - Cauchy-Schwarz, 78
  - discrete Hölder, 236
  - discrete Minkowski, 236
  - Gagliardo-Nirenberg, 169
  - Gronwall, 237
  - Hölder, 235
  - interpolation, 236
  - Jensen, 234
  - Minkowski, 78, 235
  - Morrey, 164
  - Poincaré, 156, 178
  - Young, 234
- inner product, 78
  
- Jensen, 234
  
- kernel of an operator, 17
  
- Lax-Milgram, 91
- Lebesgue, 222, 224
- Lebesgue point, 225
- limit
  - pointwise, 62
  - weak, 33
- linear combination, 15
- linear semigroup, 118
- Lipschitz, 115, 219
  
- map
  - bilinear, 73
  - continuous, 13
  - open, 63
- matrix, 18
  - positive definite, 2
- measure, 222
  - Dirac, 141
- metric space, 218
- mild solution, 136
- Minkowski, 78, 235
- mollification, 146, 228
- monotone convergence, 224
- Morrey, 163

- multi-index, 143, 149
- net smoothness, 172
- Neumann's boundary condition, 216
- norm, 11
  - equivalent, 20
  - Hölder, 56
- open covering, 219
- operator
  - adjoint, 66, 69, 198
  - backward Euler, 124, 126
  - bounded, 17
  - closed, 64, 123
  - compact, 68, 69, 93, 102, 106, 109
  - continuous, 17
  - diagonal, 18, 38
  - differential, 19
  - elliptic, 196
  - elliptic homogeneous, 190
  - integral, 20, 70
  - linear, 16
  - multiplication, 20
  - partial differential, 143
  - Picard, 116
  - positive definite, 89
  - resolvent, 126
  - shift, 19
  - symmetric, 107, 109
  - uniformly elliptic, 186
- ordinary differential equation, 115
  - in a Banach space, 115
  - linear, 118
- orthogonal projection, 80
- orthogonality, 80
- orthonormal basis, 85, 88
- orthonormal set, 84
- parabolic equation, 200
- partial differential equation
  - elliptic, 185
  - hyperbolic, 207
  - parabolic, 200
- partial differential operator, 143
- partition of unity, 232
- perpendicular projection, 80, 99
- Pettis, 227
- Picard, 116
- Poincaré, 156, 178
- point spectrum, 106
- positively invariant set, 136
- product space, 64
- Rademacher, 181
- range of an operator, 16
- Rellich-Kondrachov, 175
- resolvent identities, 126
- resolvent integral formula, 127
- resolvent operator, 126
- resolvent set, 106, 126
- Riesz, 82
- Schmidt, 109
- Schwarz, 78
- semigroup, 201, 205
  - contractive, 122
  - generation of, 128
  - of linear operators, 120
  - of type  $\omega$ , 121
  - strongly continuous, 121
- semigroup property, 121
- semilinear equation, 136
- seminorm, 24
- separable space, 218
- separation of convex sets, 31
- sequence, 13
  - Cauchy, 13, 218
  - convergent, 218
  - weakly convergent, 67
- series, 13
  - Fourier, 88
  - orthogonal, 87
- set
  - closed, 218
  - compact, 219
  - compactly contained, 152
  - connected, 218
  - convex, 233
  - dense, 218
  - measurable, 222
  - open, 218
  - orthonormal, 84
  - partially ordered, 217
  - positively invariant, 136
  - precompact, 219
  - relatively compact, 219
- sigma-algebra, 222
- Sobolev, 151
- Sobolev conjugate exponent, 169
- solution
  - classical, 187
  - weak, 188
- space
  - complete, 13, 26, 218
  - dual, 32, 66

- Euclidean, 14
- finite-dimensional, 14, 20
- Fréchet, 25
- Hilbert, 79
- Hilbert-Sobolev, 152
- Hölder, 56
- Lebesgue, 225
- locally compact, 22
- metric, 26, 218
- normed, 12, 13
- of bounded continuous functions, 45
- of bounded linear operators, 17
- of continuous functions, 14, 45
- of sequences, 15
- orthogonal, 80
- reflexive, 33
- separable, 218
- Sobolev, 151
- spectrum, 106
  - of a compact operator, 106
  - of a symmetric operator, 108
- Stone-Weierstrass, 48
- strong convergence, 33
- subalgebra, 47
- support, 139
- theorem
  - a.e. differentiability, 179
  - Ascoli, 54, 59
  - Baire category, 62, 221
  - Banach-Alaoglu, 34
  - Banach-Steinhaus, 61
  - Bochner, 227
  - closed graph, 64
  - contraction mapping, 115, 219
  - Dini, 46
  - dominated convergence, 224
  - Egoroff, 223
  - Fredholm, 101
  - Fubini, 226
  - Gagliardo-Nirenberg embedding, 172
  - Hahn-Banach, 27
  - Hilbert-Schmidt, 109
  - Lax-Milgram, 91
  - Lebesgue, 225
  - monotone convergence, 224
  - Morrey embedding, 168
  - open mapping, 63
  - Pettis, 227
  - Rademacher, 181
  - Rellich-Kondrachov compactness, 175
  - Riesz representation, 82
  - Sobolev embedding, 172
  - Stone-Weierstrass, 48, 111
  - uniform boundedness principle, 61
  - weak convergence, 33, 92
  - weak derivative, 140, 144, 156
  - weak limit, 33
  - weak solution, 198
  - weak star convergence, 33
- Young, 234