
Index of notation

- $\alpha_S^n(i)$ signature map on $H^n(\mathcal{O}_S)$, **570**
 $A(G)$ Burnside ring of G , **71**
 $A(X)$, $A_n(X)$ K -theory of spaces, $K(\mathcal{R}_f(X))$, **369**
 $A^{fd}(X)$ K -theory of finitely dominated spaces, **370**
 $\mathbf{Az}(R)$ category of Azumaya algebras, **116**
 $B(F)$ Bloch's group for a field, **536**
 BC geometric realization of a category, **313**
 BC^{top} geometric realization of a topological category, **321**
 BG^δ classifying space of a discrete group, **524**
 BG_ε subcomplex of BG^δ , **524**
 $BGL(R)^+$ connected K -theory space of R , **285**
 B_k Bernoulli numbers, **519**
 BO classifying space for real vector bundles, **92**
 BO_n classifying space for real vector bundles, **42**
 BSp classifying space for symplectic vector bundles, **92**
 BSp_n classifying space for symplectic vector bundles, **42**
 BU classifying space for complex vector bundles, **92**
 BU_n classifying space for complex vector bundles, **42**
 $C(R)$ cone ring of R , **6**
 C/d or $d \setminus C$ comma category, **314**
 $\text{Cart}(R)$ Cartier divisor group, **22**
 $\text{Cart}(X)$ Cartier divisors on X , **61**
 $\mathbf{Ch}(\mathcal{A})$ chain complexes in \mathcal{A} , **176**
 $\mathbf{Ch}^{hb}(\mathcal{A})$ homologically bounded complexes, **420**
 $\mathbf{Ch}_{\text{pcoh}}^{hb}$ pseudo-coherent complexes, **421**
 $\mathbf{Ch}_{\text{perf}}(R)$, $\mathbf{Ch}_{\text{perf}}(X)$ perfect chain complexes, **420**

- $\mathbf{Ch}_S^b \mathbf{P}(R)$ bounded S -torsion complexes, **185**
 $CH^i(R)$ generalized Weil divisor class group, **133**
 $\mathrm{Cl}(R)$ Weil divisor class group of R , **26**
 c_n Chern classes, **107**
 $D(R)$ Weil divisor group, **25**
 $E(R)$ elementary group, generated by elementary matrices, **198**
 \mathcal{EA} extension category, **359**
 $\mathrm{End}_*(k)$ K -theory of endomorphisms, **354**
 $\mathbf{End}(R)$ category of endomorphisms, **144**
 F_{-1} contraction of F , **230**
 $\mathbf{FP}(R)$ faithfully projective R -modules, **116**
 $\mathbf{F}(R)$ category of based free modules, **327**
 $\mathbf{Free}(R)$ category of free modules, **144**
 $G(R), G(X)$ K -theory of finitely generated/coherent modules, **350**
 $G(R \text{ on } S)$ relative G -theory for $R \rightarrow S^{-1}R$, **419**
 $G(X \text{ on } Z)$ relative G -theory for $X \setminus Z \rightarrow X$, **419**
 $G\text{-Sets}$ category of G -sets, **115**
 $G_* \mathcal{A}$ Gillet-Grayson construction, **377**
 $G_0(R), G_0(X)$ K_0 of $\mathbf{M}(R)$, of $\mathbf{M}(X)$, **126**
 $G_0^{\mathrm{der}}(X)$ G_0 of pseudo-coherent modules, **187**
 $GL_n(I)$ linear group of a nonunital ring I , **6**
 $GL_n(R)$ group of invertible $n \times n$ matrices, **2**
 $GL(R)$ linear group of a unital ring, **197**
 Grass_n Grassmann manifold, **41**
 $GW(F)$ Grothendieck-Witt ring, **118**
 \mathbb{H} quaternion algebra over \mathbb{R} , **523**
 $\mathbb{H}_{\mathrm{zar}}(-, A)$ Zariski descent spectrum, **489**
 $\mathbf{H}(R)$ R -modules with finite resolutions, **148**
 $\mathbf{H}_S(R)$ S -torsion modules in $\mathbf{H}(R)$, **149**
 $\mathbf{H}(X)$ \mathcal{O}_X -modules with finite resolutions, **160**
 H_0 ring of continuous maps $X \rightarrow \mathbb{Z}$, **77**
 $\tilde{H}^2(R; \mathbb{Z}_2(i))$ subgroup of $\tilde{H}^2(R; \mathbb{Z}_2(i))$, **577**
 HC_* cyclic homology, **440**
 $\mathbf{H}_Z(X)$ modules in $\mathbf{K}(X)$ supported on Z , **170**
 $I \int X$ translation category, **315**
 IBP invariant basis property, **2**
 $\mathrm{iso} S$ category of isomorphisms in S , **327**
 $j(R)$ signature defect of R , **573**
 $\mathbf{K}^B(R), \mathbf{K}^B(X)$ Bass K -theory spectrum, **383**
 $K(\mathcal{A}) = \Omega BQ\mathcal{A}$ Quillen K -theory space, **350**
 $K(\mathcal{C}) = \Omega BwS\mathcal{C}$ Waldhausen K -theory space, **368**
 $\hat{\mathbf{K}}(R)_\ell$ ℓ -adic completion of \mathbf{K} , **309**

- $K(R \text{ on } S)$ relative K -theory for $R \rightarrow S^{-1}R$, **420**
 $K(X \text{ on } Z)$ relative K -theory for $X \setminus Z \rightarrow X$, **439**
 $KH(R), KH(X)$ homotopy K -theory of R or X , **394**
 $K_0(\mathcal{A})$ K_0 of an abelian category, **124**
 $K_0(\mathcal{C})$ K_0 of an exact category, **141**
 $K_0(w\mathcal{C})$ K_0 of a Waldhausen category, **173**
 $\widetilde{K}_0(R)$ ideal of $K_0(R)$, **78**
 $K_0(R)$ K_0 of a ring, **74**
 $K_0(R \text{ on } S)$ K_0 of S -torsion homology complexes, **185**
 $K_0^\square(S)$ K_0 of a symmetric monoidal category, **114**
 $\widetilde{K}_0(X)$ ideal of $K_0(X)$, **159**
 $K_0(X)$ K_0 of a scheme, **142**
 $K_0^{\text{der}}(X)$ K_0 of perfect modules, **188**
 $K_G^0(X)$ K_0 of topological G -bundles, **117**
 $K_1(R)$ K_1 of a ring, **198**
 $K_2(R)$ K_2 of a ring, **237**
 $K_3^{\text{ind}}(F)$ $K_3(F)/K_3^M(F)$ (K_3 -indecomposable), **536**
 $K_n(\mathcal{A})$ K_n of an exact category, **350**
 $K_{\mathbb{Q}}^{(i)}$ eigenspace in λ -ring K , **107**
 $K_n^{(i)}(R)$ eigenspace in $K_n(R)$ for ψ^k , **346**
 $K_n(R)$ K_n of a ring, **285**
 $K_{-n}(R)$ negative K -groups of R , **229**
 $K_n(R, I)$ relative K -groups of an ideal, **293**
 $K_n(R; \mathbb{Z}/\ell)$ K_n with coefficients, **306**
 $K_n^\square(S)$ K_n of a symmetric monoidal category, **329**
 $K_n(X)$ K_n of a scheme, **351**
 $\widetilde{KO}(X)$ reduced K -theory, **90**
 $KO(X)$ K -theory of real vector bundles, **89**
 $KO^0(X), KO^n(X)$ representable KO -theory, **92**
 $KSp(X)$ K -theory of symplectic vector bundles, **89**
 $KSp^0(X), KSp^n(X)$ representable KSp -theory, **92**
 $KU(X)$ K -theory of complex vector bundles, **89**
 $KU^0(X), KU^n(X)$ representable KU -theory, **92**
 KV_n Karoubi-Villamayor groups, **386**
 LF contraction of F , **230**
 $\mathbf{M}(R)$ finitely generated R -modules, **126**
 $\mathbf{M}^i(R)$ modules supported in codimension $\geq i$, **478**
 $\mathbf{M}_S(R)$ S -torsion R -modules, **128**
 $\mathbf{M}_{gr}(S)$ category of graded S -modules, **138**
 $\mathbf{M}(X)$ category of coherent modules, **127**
 $\mathbf{M}_Z(X)$ coherent modules supported on Z , **130**

- $M^{-1}M$ group completion of a monoid, **69**
 M_n monomial matrices in $GL_n(F)$, **545**
 $M_n(R)$ ring of $n \times n$ matrices, **2**
 $\mathbf{mod}_S(R)$ category of S -torsion modules, **132**
MR Mumford-regular vector bundles, **163**
 $\mu^{\otimes i}$ twisted Galois representation, **303**
 $\mathrm{Nil}(k)$ K -theory of nilpotent endomorphisms, **354**
 $\mathbf{Nil}(R)$ category of nilpotent endomorphisms, **145**
 $NK_n(R)$ the quotient $K_n(R[t])/K_i(R)$, **222**
 $NS(X)$ Néron-Severi group, **68**
 $\nu(n)_F$ logarithmic de Rham group, **273**
 ΩG loop space of G , **90**
 $\Omega(BG)$ loop space of BG , **42**
 Ω_F^n Kähler differentials, **266**
 ΩR algebraic loop ring of ring R , **391**
 $\mathcal{P}(F)$ scissors congruence group, **536**
 $\mathbf{P}(R)$ category of projective modules, **9**
 $\pi_1(BC)$ fundamental group of a category, **316**
 $\pi_3^{\mathrm{ind}}(BM^+)$ indecomposables of $\pi_3(BM^+)$, **547**
 $\pi_n(X; \mathbb{Z}/\ell)$ homotopy with coefficients, **304**
 $\mathbf{Pic}(R)$ Picard category (line bundles), **115**
 $\mathrm{Pic}(R)$ Picard group of R , **20**
 $\mathrm{Pic}(X)$ Picard group of X , **55**
 $\mathrm{Pic}_+(R)$ narrow Picard group, **573**
 \mathbb{P}^n projective n -space, **55**
 QA Quillen's Q -construction, **348**
 $\mathbf{Quad}^\epsilon(A)$ category of quadratic modules, **328**
 $\mathbf{Quad}(F)$ category of quadratic spaces, **120**
 ρ rank of $K_4^M(F) \rightarrow K_4(F)$, **576**
 $R(G)$ representation ring of G , **72**
 r_1, r_2 number of real (complex) embeddings, **297**
 $R[\Delta^\bullet]$ simplicial ring of standard simplices, **386**
 $\mathbf{Rep}_\mathbb{C}(G)$ category of complex representations of G , **115**
 $\mathcal{R}_f(X)$ finite spaces over X , **174**
 $\mathcal{R}_{\mathrm{fd}}(X)$ finitely dominated spaces over X , **186**
 R^n free R -module of rank n , **1**
 $\sigma(M)$ shift automorphism on graded modules, **138**
 Σ_n symmetric group of permutations, **287**
 $S^{-1}S$ group completion category, **328**
 $\mathrm{Seq}(F, R)$ sequence for contracted functors, **230**
 $\mathbf{Sets}_{\mathrm{fin}}$ category of finite sets, **115**
 $SK_0(R)$ ideal of $K_0(R)$, **81**

- $SK_0(X)$ ideal of $K_0(X)$, **159**
 $SK_1(A)$ subgroup of $K_1(A)$, A semisimple, **200**
 $SK_1(R)$ subgroup of $K_1(R)$, **198**
 $SL_n(R)$ special linear group of a ring, **198**
 (S_n) stable range condition, **5**
 $S_n\mathcal{C}$ category of n -fold extensions, **366**
 $sr(R)$ stable range, **5**
 $St(R)$ Steinberg group, **237**
★ star operation on $St(R)$, **245**
 $\langle S, X \rangle, S^{-1}X$ localization categories, **333**
 $U(R)$ group of units in R , **2**
 $Unip(R)$ group of unipotent matrices, **385**
 v_1^4 generator of $\pi_8^s(S^0; \mathbb{Z}/16)$, **534**
 $\mathbf{VB}(X)$ category of algebraic vector bundles, **50**
 $\mathbf{VB}_{\mathbb{C}}(X)$ category of complex vector bundles, **35**
 $\mathbf{VB}_{\mathbb{R}}(X)$ category of real vector bundles, **35**
 $\mathbf{VB}_n(X)$ vector bundles of rank n , **42**
 $w_{ij}(r)$ special element of $St(R)$, **246**
 $W(F)$ Witt ring of quadratic forms, **118**
 $W(R)$ a subgroup of units, **202**
 $W(R)$ ring of big Witt vectors, **101**
 $Wh_0(G)$ 0th Whitehead group, **79**
 $Wh_1(G)$ 1st Whitehead group, **207**
 $Wh_n(G)$ n th Whitehead group, **292**
 w_i Stiefel–Whitney classes, **44**
 $w_i(F)$ exponent of the e -invariant, **516**
 $WQ(F)$ K_0 of split quadratic forms, **121**
 $wS\mathcal{C}$ Waldhausen construction, **367**
 $[X, \mathbb{N}]$ continuous maps from X to \mathbb{N} , **36**
 $\zeta_F(s)$ Riemann zeta function, **520**

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