
Index

In this index, references to the definitions of terminology are printed using **boldface** page numbers. References to primary statements or proofs of theorems are printed using *italic* page numbers.

- A, B, C* Property, 320
- Abelian group, **459**
 - cyclic, **460**
 - direct product, **460**
 - divisible, **461**
 - finitely generated, **460**
 - quotient, **460**
 - torsion, **460**
 - torsion-free, **460**
- absorbs, **470**
- Ackermann function, 39
- activation temperature, **360**
- active, **342**, 359, **360**, 364, 387
- adjoint, **270**
- affine stratification, 269
- algebraic periodic, 269
- algorithmic combinatorial game theory, 44–46
- all-moderate, **405**, 406, 412
- all-small, 63, **405**, 406, *see also* dicotic
- alternating game, **82**
 - in misère play, 278
- annihilation game, 214, 489
- antichain, 154, 167, **463**
- Archimedean Principle, 71, 156, 403
 - infinitesimal analogue, 85, 156
- arithmetic periodic, **187**, 194, 196–197, 203
- atomic, **142**, 143–144, 149
- atomic value, **151**
- atomic weight, 97, 136, **142**, 140–146
 - and liberty counts in Go, 49
 - galvanized, **150**
 - of a stopper, 298–299, 307
 - of HACKENBUSH positions, 149
 - transfinite, **409**
- atomic weight calculus, 144–146
- Austin’s Theorem, 196
- Avoider–Enforcer convention, *see* play convention, reverse weak win
- Bach’s Carousel, **315**, 316, 319
- balloon trajectory, **367**, 368
- base, **365**
- Berlekamp’s Sign-Expansion Rule, 421
- bigraph, **281**
 - negative, **281**
- binary game, **277**
- Binary Normal Form Theorem, 468
- Birkhoff’s Theorem, 465
- birthday, **60**, 61–62, 71, 153, **400**, 421
 - formal, **61**, **398**
 - of a number, 72–73
- board (positional game), 42
- Boolean algebra, *see* lattice, Boolean
- Bouton’s Theorem, 3, 4, 20, 35, 85, 180, 224, 236, 254, 438, 491
 - Misère Version, 225
- bynumber, **149**
- bypass, **65**
- canonical, 336
- canonical form, 64, **66**, **67**, 68
 - for finite loopy games, 317
 - for stoppers, **305**

- misère, **245**, 249
- partizan misère, **274**, 275–276
- quasi-, *see* quasi-canonical form
- reduced, *see* reduced canonical form
- subpositions of, 175
- uniqueness, 67
- Cantor Normal Form Theorem, 425, 468
- cave, **365**, 367
 - point inside, **365**
- cave temperature, **365**, 366, 369, 377
- ceiling, **24**
- chain, **463**
- children, **62**, 166–167
- chimney, **365**
- closed, **168**, **252**
- closure, **168**
- coalition, 43
 - stable, 43
- code digit, **188**, 189
- cold, **112**, 361
- combinatorial game, **1**, 8
 - positional, *see* positional game
- commensurate, **424**
- common, **192**
- common coset, **192**
- companion, **163**, 162–165
 - atomic weight of, 165
- complementary (sets of integers), **199**
- compound, *see* sum
- confused with, **58**
- confusion interval, **75**, 76, 81
 - endpoints, 75
- consecutive move banned, *see* alternating game
- constraint logic, 44
- convergence conjecture (WYTHOFF), 206
- converges, **432**
- Conway Normal Form Theorem, 426
- Conway product, **412**, 423
 - impartial, 220, 439
- Conway's Cancellation Theorem, 248
- cooled by, **102**
- cooling, 101–102, 109–111
 - homomorphism, 109
 - monotonic, 109–110
 - well-defined, 107
- coupon, 335
- coupon stack, 335, **336**
 - Left and Right stops, 336
- critical temperature, **116**, 122
- cycle, **281**
 - almost monochromatic, 307
 - monochromatic, **281**, 306
 - tame, **309**
 - wild, **309**
- cycle pattern, **309**
- \mathcal{D} -position, 207, **209**
- dead end, **277**, 278
- Dedekind cut, 400–401
- degree, **322**, 323–326
 - stable, **322**
- degree of loopiness, *see* degree
- Desirability Theorem, 100
- dicotic, **60**, 141, 147, 299, 331
 - atomic weight of, 144–148
 - born by day n , 157
 - canonical form of, 68
 - disjunctive sums, 60
 - HACKENBUSH positions, 83
 - in misère play, 277–278
 - is infinitesimal, 83
 - lattices of, 166
 - origin of terminology, 63
 - transfinite, 400, 404
- Dicotic Avoidance Theorem, 135
- Dicotic Translation Theorem, 98
- discriminant, **233**, 236
- disjoint, 5, 10
- distinguish, **244**
- divisible, 167, 169–170
- dogmatic, 383, 396
- domain
 - of a sign sequence, *see* length
- dominated, 19, **64**, 65–68, 290, 301
 - misère, **271**, 272–273
 - onside-, **320**
 - strongly, **289**
- Dominated Incentives Theorem, 62, 78, 87, 336
- dormant, **342**, 359, **360**, 364, 392
- double-up (\uparrow), **59**
 - exceeds *, 59
- down (\downarrow), **59**, 85
- downlinked, **274**
- downsum, **318**, 319–320
- Downsum Absorbancy Rule, 324, 326
- draw, 352
- dyadic rational, **69**
- edge
 - of a game tree, 60
- empty game, **15**, **54**, 56
- end, **270**, 271, 275, 277
- enriched environment, 334–335
- enumeration of G_n
 - bounds on, 166
- epsilon number, 437, **468**
 - generalized, **437**
 - quasi-, **437**
- equality as a defined relation, 12
- even, **68**
- even-tempered, **130**
- evil, **192**

- excludent, **181**
 far star, *see* remote star
 Ferguson's Pairing Property, *195*
 fickle, **232**, 231–233, 254, 262
 finite, 9, **34**, 53, **208**
 loopy game, **281**
 firm, **232**, 231–233, 254, 262
 floor, **24**
 flower, **91**, 92–93, 98, 136
 blossom, **92**
 stem, **92**
 flower garden, **91**, 92, 136–137, 139–140, 144
 flowering tree, **98**
 forcing pass, 357, 370, 392
 Forcing Pass Lemma, *371*
 formal birthday, 68
 fullstop, **339**, 340–341, 354, 362
 with koban, **355**
 fundamental equivalence, 11–14, 40, 55, 139, 207, 225, 249, 283, 310, 399
 Fundamental Theorem of Combinatorial Game Theory, 9, 46, 55, 209
 Long Form, *398*
 fusion, 304–305, 307, 321
 Fusion Lemma, *304*, 307
 fuzzy, **58**

G-value, *see* nim value
 game tree, **60**, 61, 65, 86, 97
 game value, **12**, 20, *see also* nim value
 misère, **225**, 249–250
 normal-play short partizan, 56
 partizan loopy, **283**
 reduced, *see* reduced game value
 transfinite, **399**
 Γ -heap, **184**
 genus, **231**, 236, 241–242, 249, 253–254, 262
 addition table, 233
 extended, **239**, 241–242
 generalized, **242**
 restive, **238**
 tame, **238**
 Gift Horse Principle, *63*
 Golay code, **183**
 golden ratio, 198, 205
 ground (HACKENBUSH), **6**
 group, *see* Abelian group
 group structure
 of \mathbb{G}^1 , 181
 of \mathbb{G}_n , 168
 of \mathbb{G} , 172–177
 Grundy value, *see* nim value
 Grundy's conjecture, 248

 Hamming code, **183**
 Hamming distance, **183**
 Hasse diagram, 167, **462**
 heap, **184**
 heap game, **184**, 195, 251
 heated by, **112**
 heating, 112–113
 depends on form, 113
 height
 of a plumtree, 320
 Hensel's Lemma, 436
 hereditarily closed, **168**
 hereditary structure
 extreme values of \mathbb{G}_n , 156–157
 of \mathbb{G}_2 , 154
 of dicotic games, 157
 of reduced games, 157
 hexadecimal game, **194**, 195, 197
 hill temperature, **365**, 366, 369
 hot, **112**, 127–128
 hotstrat, 345–346
 hyperactive, **382**, 383, 396

 idempotent, 261, **470**
 impartial, 6, **34**, 179, 397, 400
 loopy game, **208**, 207–213, **281**
 incentive, **62**, 63, 150
 necessarily $\triangleleft 0$, 62
 negative, 69, 81
 of a canonical form, 68
 of a game of finite order, 171, 173
 of a noninteger, 80
 of a nonnumber, 78
 of a number, 69, 72
 of a reduced canonical form, 134
 of $G \cdot \uparrow$, 145
 temperature of, 121
 Inf-dominated, **126**, 125–127
 Inf-Replacement Lemma, 125, 129
 Inf-reversible, **126**, 125–127
 Inf-senseless, 135, 351
 Inf-sensible, 135
 Inf-simplest form, *see* reduced canonical form
 Inf-Simplest Form Theorem, 128
 infimum
 of a set of game values, **291**, 307
 infinitesimal, 20, **83**, 82–97, 102, 280
 long game, **404**
 relative, 84
 stops, 84
 infinitesimally close, **84**, 102, 124
 input complexity, **36**
 Integer Avoidance Theorem, *80*, 141
 integers
 as a subgroup of \mathbb{G} , 59
 intensity, *see* thermal intensity

- Intermediate Value Theorem, 105, 106–108
interval, **71**, 105
 closed, **105**
interval notation, 71
intractable, 44
inverse
 of a surreal number, **416**
ish, **84**
isomorphic, 12–13, 20
- join, **463**
join-irreducible, 166, 411, **464**, 465
 elements of \mathbb{G}_n , 161–162
junction point, **105**, **365**, 367
- kernel, **261**, 263, 268
ko, 351, **362**, 365
 cold, 361
ko adjustment, **388**
ko option, **362**, 370
koban, 352, 354, **355**, 377–378
komaster, 381
- \mathcal{L} -position, **7**
lattice, 159, **463**, 464, 473
 Boolean, **464**
 distributive, 159, **463**, 464–465
 transfinite, 410–411
lattice game, 269
lattice structure
 distributivity of \mathbb{G}_n , 160–161
 of \mathbb{G}_n^0 , 166
 of \mathbb{G}_n/Inf , 166
 symmetries of \mathbb{G}_n , 162–165
Lawnmower Theorem, 83, 84, 404, 406
Left, **1**
Left edge, **281**
length
 of a sign sequence, **418**
 of an octal game, **189**
Lessons in Play, 25
lexicode, **183**, 221
Lexicode Theorem, 183
line segment, **105**
linked, **244**
lonely, **163**, 165
long game, 397, **398**, 410
loopfree, 28, **34**, 53, **282**
loopy, 28, 32, **34**, 279
loopy game, **281**, 334
 complex, 363–364, 377
 fixed, **321**
 free, **321**
 simple, 354, **362**
 transfinite, 411
- Maker–Breaker convention, *see* play
 convention, weak win
mast, **105**, 366–367
 crooked, 366
mast value, 104, **105**, 107, 109, **360**
 equal to mean, 111
 komaster, **382**
mate, **243**, 247, 270
Mathematical Go, 30
mean, 78, **79**, 101, 113
 additive, 79
mean value, *see* mean
Mean Value Theorem, 79, 82, 122
meet, **463**
meet-irreducible, **464**
mex, **180**
mex function, **265**, 267–268
Mex Interpolation Principle, 268
mex rule, 180, 181, 185, 227, 256
 generalized, 266
 misère, 228
meximal set, **266**
minimal excluded value, *see* mex
miny- G (\neg_G), *see* tiny- G
mirror-image strategy, **2**, 57, 62, 64, 286
misère canonical form, *see* canonical form
misère play, 4, **34**, 223
misère quotient, **250**, 259
 finite, 251, 259
 infinite, 269
 nontrivial, **259**
 normal, **263**, 268
 of small order, 260
 partial, **253**
 partizan, 276
 regular, **268**
 tame, **254**, 253–255, 262
Mock Turtle Theorem, 183
moderate, **405**, 406, 412, *see also*
 all-moderate
monoid, 250, **470**, *see also* semigroup
 bipartite, **258**
 free, **471**
 partially ordered, 276
 reduced bipartite, **258**
Monte Carlo algorithm, 47
mutual divisibility, **470**
- \mathcal{N} -position, **7**, **209**
negative, 17, **54**, 270
 of a loopy game, **282**
neutral threat environment, 396
nim arithmetic, 397
nim value, 2, 16, 20, **180**, 179–182, 214,
 224, 254
 loopy, **207**, **212**, 213–214
 misère, **229**, 230–231

- of a heap game, 184–185
- of DAWSON’S KAYLES, 186, 191
- of GRUNDY’S GAME, 185
- of KAYLES, 191
- of WYTHOFF, 200
- transfinite, 438
- nim-addition rule, 85, 180–181, 184–186, 228, 232, 477
- misère, 228
- transfinite, 438
- nim-heap, 20, 84
- nim-multiplication rule, 220
- transfinite, 439
- nim-product, 215, 214–220
- transfinite, 439
- nim-root, 440
- nim-sum, 181, 214
- of flower stems, 92
- transfinite, 438
- number, 84, 138, 179, *see also* remote star
- transfinite, 408, 409, 438
- 9-dan stumping problem, 29, 123
- Noah’s Ark Theorem, 241
- normal form
- for integers, 216
- normal play, 4, 34, 223
- Norton product, 141
- generalized, 150
- transfinite, 408
- transfinite generalized, 411
- Norton’s Lemma, 91, 296
- number, 18, 69, 68–71
- adorned, 81
- canonical form of, 70
- real, *see* real number
- surreal, *see* surreal number
- Number Avoidance Theorem, 72, 126
- failure for long games, 404
- strong form, 73, 78
- Number Translation Theorem, 78, 110
- failure for long games, 404
- number tree, 73
- transfinite, 421
- numberish, 81, 84

- octal game, 188, 189–192, 196
- odd, 68
- odd-tempered, 130
- odious, 192
- offside, *see* sides
- offside approximation, *see* sidling
- approximation
- omega (ω), 32, 398, 466
- ω -power, 424, 425, 437
- omnific integer, 436
- On Numbers and Games*, 25
- ONAG, *see* *On Numbers and Games*

- onside, *see* sides
- onside approximation, *see* sidling
- approximation
- option, 8, 53, 180, 208
- of a loopy game, 282
- order
- finite, 171
- odd, 167, 409
- order-isomorphic, 465
- ordertype, 465
- ordinal, 398, 403, 466
- limit, 466
- normal form, 468–469
- signed, 404, 406
- successor, 466
- ordinal sum, 296
- orthodox, 334, 336, 342, 351
- at temperature t , 341, 347, 351
- orthodox accounting, 383
- Orthodox Accounting Theorem, 347, 348, 360, 362, 378, 387–388, 391
- orthodox forecast, 343–344, 347–348, 383
- for simple loopy games, 387
- Orthodox Forecast Theorem, 343, 345
- orthodox play, 344
- orthodoxy, 360
- outcome, *see* outcome class
- biased, 309
- outcome class, 3, 6, 14, 55, 57–58, 209, 399
- loopy, 282–283, 299, 310
- overheated by, 114
- overheating, 114–115
- generalized, 115, 122

- \mathcal{P} -portion, 250, 259
- \mathcal{P} -position, 7, 56, 209
- adjoining as moves, 206
- of WYTHOFF, 198, 200, 206
- partial order, 462
- on \mathbb{G} , 57
- on loopfree outcome classes, 57
- partially ordered set, *see* poset
- particle, 116
- partizan, 6–7, 34
- strictly, 35
- partizan subtraction game, 123
- period, 187
- minimal, 187
- periodic, 187, 188, 190
- arithmetic, *see* arithmetic periodic
- purely, 187
- Periodicity Theorem
- Generalized, 196
- Misère, 248
- Octal, 190, 191, 235, 256
- Quotient, 257, 258
- Subtraction, 188

- Tame, 235
 placid, **382**, 388, 391
 planar, 167
 play, **9**, 27
 σ -, **285**
 according to σ , **285**
 play convention, 4, **34**
 misère, *see* misère play
 normal, *see* normal play
 reverse weak win, 42
 strong win, 42
 weak win, 42, 49
 plumbtree, **314**, 315, 317, 319–320
 grafting, 320
 height, 320
 poset, 159, **462**, 463
 homomorphism, 462
 ideal, **464**
 isomorphism, 462
 position, **1**
 positional game, 42
 preperiod, **187**
 minimal, **187**
 product
 natural, **469**
 proviso, **226**, 228, 248
 pseudonumber, **300**
 surreal, *see* surreal pseudonumber
 Pseudonumber Avoidance Theorem, 300
 Pseudonumber Translation Theorem, 300
 pseudoptimal, **300**
- \mathcal{Q} -position, 43–44
 quasi-canonical form, **170**, 171, 173
 quenching temperature, **377**
 quotient map, 250
- \mathcal{R} -position, **7**
 r.b.m., *see* monoid, reduced bipartite
 rank, 207, **212**, 214
 rare, **192**
 rational
 dyadic, *see* dyadic rational
 real number, 400–401
 real-closed, 412
 Rédei's Theorem, 472
 reduced, **129**, 157, 178
 born by day n , 157
 lattices of, 166
 reduced by, 131
 reduced canonical form, **128**, **129**, 157
 uniqueness, 128
 reduced game value, 132
 reduction, 131
 redwood bed, 44
 redwood spider, **82**
 remote, 137, **138**
 remote star, **138**, 139, 409
 remoteness, **48**, 50, 214
 misère, **48**
 repetition, 352
 local, 354
 Replacement Lemma, 64, 181
 misère, 226
 restive, **236**, 237–238, 241, 255
 generally, **239**, 241
 restless, **236**, 237–238, 241
 revenge rule, 43
 reversible, **65**, 64–68, 154, 290, 301, 410
 misère, **245**, 246
 misère partizan, **271**, 272–273
 onside-, **320**
 strongly, **289**
 reverting move, **226**
 Right, **1**
 Right edge, **281**
 ruler regularity, 197
 ruleset, 8, **36**, 44
 dual, 207
 impartial, 195
 reflexive, 207
 run, **9**, 27
 alternating, **9**, 289
 infinite, 352
 swivel, *see* swivel run
- saltus, **187**, 194, 196
 sapp regularity, 197
 scaffold, 104, **106**, 364, **365**, **369**, 370
 score, 75, **103**, 105, 335, 344, 365
 biased, **387**
 enriched, **337**, **339**
 enriched, with koban, **355**
 komaster, **381**
 komonster, **392**
 of a loopy game, **356**
 of an alternating run, **345**
 section, **410**
 numeric, **410**
 semigroup, **470**, *see also* monoid
 Archimedean component, **472**
 congruence, **470**
 finite, 472–473
 finitely generated, **471**
 finitely presented, **472**
 free, **471**
 kernel, **473**
 presentation, **471**
 quotient, **470**
 senseless, **68**, 351
 sensible, **68**, 351
 sentestrat, 347–350, 388
 series
 transfinite, 426

- short, **9**, 26–28, **34**, 53, 282
short game, **54**, 397
Sibert–Conway Decomposition, 259
sides, 309, **310**, 314, 317, 393
 of a plumtree, 315
 of a stopper, 313
 uniqueness, 313
sidling approximation, **292**, 293–296, 317
Sidling Theorem, 317, 321
sign expansion, **418**, 421
sign sequence, **418**
 restriction, **418**
signature, **182**
similar, **295**, 303–304
 for n moves, **295**
Simplest Extension Theorem, 440, 444, 452
simplest form, *see* canonical form
Simplest Form Theorem, 67, 68, 248, 301, 305
 for stoppers, 306
 misère, 246
 misère partizan, 276
Simplicity Theorem, 72, 115, 403, 417
simplifies to, **226**, 227, 246
small, **405**, 408–409, *see also* all-small
solution, **36**, 35–39, 44
solved, 4, **36**, 37–38, 186
 ultra-weakly, **38**
 weakly, **38**
sparse space, **192**, 193–194
 for GRUNDY’S GAME, 192
 for KAYLES, 192
spider, **82**
Sprague–Grundy Theorem, 180, 179–181, 183, 438
Sprague–Grundy theory, 279
 generalizations of, 249–250
Stability Conjecture, 328
stable, **328**, 343–344
stable degree, **322**
stalk (HACKENBUSH), 122
star m ($*m$), *see* nim-heap, *see* number
star $(*)$, 20, 67, 102
 as an ordinal sum, 93
 order of, 59
 $*$ -projection, **134**
start vertex, **281**
stop, **75**, 76–78, 124, 361, *see also*
 confusion interval
 adorned, **81**
 bounds on sums, 77
 invariant of form, 76
 of a stopper, **300**
stopper, 279, **289**, 313, 321
 similarity, 303
 transfinite, 411
stopper-sided, 279, **315**, 316–317, 319–320, 322
strategy, **285**, 286
 complete survival, **289**
 concentrates, **312**
 survival, **285**, 286, 288, 290, 311–312
 winning, **285**, 287, 311
strategy-stealing argument, 38
structural constraint, 33
structure of \mathbb{G}
 as a partially ordered Abelian group, 178
Structure Theorem for Finitely Generated
 Abelian Groups, 461
subgroup, **460**
 generated, **460**
 maximal, **470**
 of a semigroup, **470**
sublattice, **464**
subposet, **462**
subposition
 proper, **9**
subposition count, **372**
subtraction game, **184**, 195–196, 214
 all-but, **196**
 finite, **187**
 partizan, **15**, 98, *see* partizan subtraction
 game
subtraction set, **184**
sum
 conjunctive, 40, **41**, 48, 50
 continued conjunctive, **41**, 50
 diminished disjunctive, **41**, 50
 disjunctive, 10, **11**, 40, **41**, **54**
 galvanized, **150**
 natural, **469**
 ordinal, **41**, **89**, 90–91
 depends upon form, 91
 of HACKENBUSH stalks, 90
 selective, 40, **41**, 48, 50
 sequential, **41**, 51, 182, 205
 shortened selective, **41**, 50
 side, **41**
superstar, **98**
support, **428**
supremum
 of a set of game values, **291**, 307
 of a set of ordinals, **466**
surreal integration, 438
surreal number, 28, 82, 397, **401**, 402–404
 absolute value, **403**
 β^{th} approximation, **417**
 exponential, 437
 irreducible, **437**
 large, **405**, 406
 normal form, 424–432
 reducible, **437**
 square root, **423**

- surreal pseudonumber, 411
 survival move, **284**, 300
 switch, **73**
 swivel chair, 287, 328
 swivel run, **287**
- t*-plicate, 196
 take-and-break game, **194**, 196
 tame, 224, **232**, 233, 236, 249, 253
 generally, **238**, 241
 temper, 130–132, 135
 temperature, 101, **102**, 335, **360**
 ambient, 344, **345**, 347, 392
 biased, **387**
 board, **344**
 biased, **387**
 board activation
 biased, **387**
 generalized, 354
 komaster, **382**
 negative, 339–341, 362
 of $2 \times n$ AMAZONS, 121
 of a number, 102, 339
 of a sum, 109
 submaximal, 111
 temperature auction, 337
 temperature class, **112**
 temperature drop, 334, 344, **345**, 348
 tepid, **112**
 terminal, **208**
 thermal dissociation, **116**, 117–118
 existence and uniqueness of, 117
 thermal intensity, **368**, 369, **373**
 thermal shock, 344
 thermograph, **103**, 104–105, 334
 komaster, **382**
 komonster, **393**
 plotting conventions, 103
 properties of, 108
 thermographic calculus, 105–109
 for complex loopy games, 377
 for simple loopy games, 369–376
 generalized, 354
 komaster, 383–387
 thermographic intersection, 104, 365–367,
 368, 377
 analytic characterization, 369, 377
 threat, 378–381
 standard, **381**
 threat environment, **381**, 383
 neutral, 396
 three-repetition rule, 352
 tiny
 transfinite, **408**
 tiny- G (\dagger_G), 88–89, 96–97
 infinitesimal relative to \uparrow , 88
 tends to 0, 89
- torsion element, **460**
 totally ordered set, **463**
 trajectory, **105**, 106, 365
 transfinite, 32, **34**, 398
 transfinite induction, 467
 transition algebra, **266**
 transitive game, 133–134
 hereditarily, 133, 135, 149, 158–159, 167
 tree
 game, *see* game tree
 in HACKENBUSH, 21, *see also* flowering
 tree
 of numbers, *see* number tree
 truncated, **429**, 430
 turning game, **182**
 cross product, **220**
 Two-Ahead Rule, 92, 94–95
 for SUPERNIM, 98
 generalized, 144
 2-power, **181**
 Fermat, **215**, 221, 448
- UCT search, 48
 unit, **233**
 universal embedding property
 for Abelian groups, 411
 for fields, 436
 for partially ordered Abelian groups, 412
 for totally ordered fields, 436
 unraveling, 321
 unsolvable, 44
 unstable, 361
 up (\uparrow), **59**, 67, 83, 85
 canonical form, 87
 confused with $*$, 59
 exceeds $*m$, 87
 multiples of, 85
 sums with numbers, 87
 up- n^{th} (\uparrow^n), **94**
 canonical form, 94
 confused with $*m$, 95
 up-star ($\uparrow*$), 96
 as an ordinal sum, 93
 canonical form, 67
 uplinked, **274**
 upsum, **318**, 319–320
 uptimal, **95**, 93–96, 122, 135, 297–298, 324
 companion of, 165
 fractional, 99
 generalized, 99, 165
 transfinite, **408**, 410
 optimal confusion interval, **95**, 97
 optimal notation, **95**, 100
 generalized, **100**
- value, *see* game value
 variety, 323, **326**, **327**

of \uparrow^{on} , 323–324, 331
of **on**, 329

wall, 104, **106**, 364, **369**

weight, **92**, 93, 98, 136, 140
atomic, *see* atomic weight

well-ordered set, **465**
of surreal numbers, 430, 432

well-ordering, **465**

wild, 224, **232**, 233, 236, 242, 249, 258

winning move, **284**, 300

winning set, 42

Winning Ways, 25

x -based, **117**

zero position, 3, 7, **17**, 56
loopy, 210

zugzwang, 30

zugzwang game, **320**, 321

weak, **320**