Contents

Preface vii

Part 1. The Core of the Theory

Chapter 1. Examples of Hyperbolic Dynamical Systems 3
§1.1. Anosov diffeomorphisms 4
§1.2. Anosov flows 8
§1.3. The Katok map of the 2-torus 13
§1.4. Diffeomorphisms with nonzero Lyapunov exponents on surfaces 23
§1.5. A flow with nonzero Lyapunov exponents 27

Chapter 2. General Theory of Lyapunov Exponents 33
§2.1. Lyapunov exponents and their basic properties 33
§2.2. The Lyapunov and Perron regularity coefficients 38
§2.3. Lyapunov exponents for linear differential equations 41
§2.4. Forward and backward regularity. The Lyapunov–Perron regularity 51
§2.5. Lyapunov exponents for sequences of matrices 56

Chapter 3. Lyapunov Stability Theory of Nonautonomous Equations 61
§3.1. Stability of solutions of ordinary differential equations 62
§3.2. Lyapunov absolute stability theorem 68
§3.3. Lyapunov conditional stability theorem 72

Chapter 4. Elements of the Nonuniform Hyperbolicity Theory 77
§4.1. Dynamical systems with nonzero Lyapunov exponents 78
§4.2. Nonuniform complete hyperbolicity 88
§4.3. Regular sets 91
§4.4. Nonuniform partial hyperbolicity 93
§4.5. Hölder continuity of invariant distributions 94

Chapter 5. Cocycles over Dynamical Systems 99
§5.1. Cocycles and linear extensions 100
§5.2. Lyapunov exponents and Lyapunov–Perron regularity for cocycles 105
§5.3. Examples of measurable cocycles over dynamical systems 109

Chapter 6. The Multiplicative Ergodic Theorem 113
§6.1. Lyapunov–Perron regularity for sequences of triangular matrices 114
§6.2. Proof of the Multiplicative Ergodic Theorem 120
§6.3. Normal forms of measurable cocycles 124
§6.4. Lyapunov charts 128

Chapter 7. Local Manifold Theory 133
§7.1. Local stable manifolds 134
§7.2. An abstract version of the Stable Manifold Theorem 137
§7.3. Basic properties of stable and unstable manifolds 147

Chapter 8. Absolute Continuity of Local Manifolds 155
§8.1. Absolute continuity of the holonomy map 157
§8.2. A proof of the absolute continuity theorem 161
§8.3. Computing the Jacobian of the holonomy map 167
§8.4. An invariant foliation that is not absolutely continuous 168

Chapter 9. Ergodic Properties of Smooth Hyperbolic Measures 171
§9.1. Ergodicity of smooth hyperbolic measures 171
§9.2. Local ergodicity 176
§9.3. The entropy formula 183

Chapter 10. Geodesic Flows on Surfaces of Nonpositive Curvature 195
§10.1. Preliminary information from Riemannian geometry 196
§10.2. Definition and local properties of geodesic flows 198
§10.3. Hyperbolic properties and Lyapunov exponents 200
§10.4. Ergodic properties 205
§10.5. The entropy formula for geodesic flows 210

Part 2. Selected Advanced Topics

Chapter 11. Cone Technics 215
§11.1. Introduction 215
§11.2. Lyapunov functions 217
§11.3. Cocycles with values in the symplectic group 221

Chapter 12. Partially Hyperbolic Diffeomorphisms with Nonzero Exponents 223
§12.1. Partial hyperbolicity 224
§12.2. Systems with negative central exponents 227
§12.3. Foliations that are not absolutely continuous 229

Chapter 13. More Examples of Dynamical Systems with Nonzero Lyapunov Exponents 235
§13.1. Hyperbolic diffeomorphisms with countably many ergodic components 235
§13.2. The Shub–Wilkinson map 246

Chapter 14. Anosov Rigidity 247
§14.1. The Anosov rigidity phenomenon. I 247
§14.2. The Anosov rigidity phenomenon. II 255

Chapter 15. C^1 Pathological Behavior: Pugh's Example 261

Bibliography 267

Index 273