
Index

- $C^{1,1}$ local group, 36
- p -adic norm, 11
- *-homomorphism, 146

- A group is virtually nilpotent if it
 - contains a finite index subgroup
 - which is nilpotent., 21
- abstract Lie algebra, 249
- additive combinatorics, 14
- adjoint representation, 44, 85
- Ado's theorem, 250
- almost period, 179
- approximate group, 162, 212
- arrow's theorem, 147
- associated neighbourhood, 119
- asymptotic notation, xiii, 159

- baby Peter-Weyl theorem, 91
- Baker-Campbell-Hausdorff formula, 43
- Baker-Campbell-Hausdorff-Dynkin formula, 48
- Birkoff-Kakutani theorem, 101
- Borel measure, 77
- Brouwer fixed point theorem, 131
- Burnside's theorem, 235

- cancellative local group, 118
- Cartan's theorem, 54
- Cayley graph, 20
- central element, 6
- centraliser, 6
- chain rule, 31
- character, 304
- characteristic subgroup, 240

- cheap structure theorem, 210
- class function, 304
- classification of one-parameter subgroups, 57
- clopen set, 127
- closed graph theorem, 57
- cocompact subgroup, 94
- cocompact topology, 61
- cocycle, 280
- cocycle equation, 280
- cohomologous, 281
- commutator, 6, 239
- compact operator, 89
- compact-open topology, 66
- compactness of integral operators, 89
- compactness theorem, 157
- complete measure space, 313
- concretely nilpotent Lie algebra, 254
- conjugate, 239
- connected component, 128
- connected space, 127
- continuous action, 127
- convolution, 88
- coset nilprogression, 165
- coset progression, 17, 164
- countable saturation, 151

- degree, 291
- derivation, 256
- derivations and vector fields, 33
- derivative map, 31
- directional derivative, 32

- edge density, 314
 elementary set, 322
 equivariant map, 298
 escape norm, 201
 existence and uniqueness of Haar measure, 78
 exponential coordinates, 41
 exponential map, 69
 extension problem, 277
- faithful action, 127
 finite trapping, 114
 formal commutator word, 243
 formal predicate, 157
 Fourier analysis on compact abelian groups, 94
 Fourier inversion formula, 303
 Freiman's theorem, 17, 164
 Fubini-Tonelli theorem, 314
- gauge transformation, 281
 general linear group, 9
 generalised arithmetic progression, 16, 164
 Gleason lemma, 202
 Gleason metric, 55
 Gleason-Yamabe theorem for abelian groups, 96
 Gleason-Yamabe theorem for compact groups, 92
 good model, 185, 213
 Gromov's theorem, 21, 224
 Gromov-Hausdorff distance, 171
 group cohomology, 281
 group germ, 267
 group of polynomial growth, 21
- Haar measure, 77, 87
 Hadamard lemma, 46
 half-open topology, 62
 Hall-Witt identity, 240
 hard analysis, 139
 Hausdorff distance, 116, 170
 height, 308
 Heisenberg group, 9
 Helfgott-Lindenstrauss conjecture, 216
 Hilbert's fifth problem, 10, 133
 Hilbert-Smith conjecture, 289
 homogeneous space, 86
 homomorphism, 29
 Hrushovski Lie model theorem, 213
 Hrushovski's Lie model theorem, 192, 195
 Hrushovski's structure theorem, 194
 hyperreal, 149
- indicator function, xiii
 infinitesimal, 159
 infinitesimal part, 159
 inner regularity, 77
 internal function, 152
 internal set, 149
 invariance of dimension, 126
 invariance of domain, 126, 132
 inverse limit, 93
 irreducible representation, 298
 isotypic component, 300
- Jacobi identity, 249
 Jordan's theorem, 6, 233
 Jordan-Schur theorem, 233
- Keisler-Fubini theorem, 319
- lamplighter group, 191
 left-invariant vector field, 34
 Leibniz rule, 33
 Levi's theorem, 258
 Lie algebra, 33
 Lie algebra of a Lie group, 34
 Lie bracket, 33
 Lie group, 8
 Lie groups are analytic, 49
 Lie's first theorem, 40
 Lie's second theorem, 50
 Lie's third theorem, 35, 50, 270
 lifting lemma, 215
 local finiteness, 77
 local Gleason metric, 121
 local Gleason-Yamabe theorem, 121
 local group, 26
 local Haar measure, 86
 local homomorphism, 29
 local Lie group, 27
 Local Lie implies Lie, 43
 local section, 283
 local topological group, 26
 Loeb measure, 317
 Los's theorem, 154
 lower central series, 241
- Margulis-type lemma, 229
 matrix exponential, 4
 measure space, 313

- monad, 267
- Newman's first theorem, 291
- Newman's second theorem, 292
- nilpotent Lie algebra, 251
- nilprogression, 165
- nilradical, 257
- no small subgroups, 106
- noncommutative progression, 18, 165
- nonstandard finite set, 158
- nonstandard function, 152
- nonstandard object, 149
- nonstandard space, 149
- nonstandard universe, 149
- normal Sanders lemma, 182
- normal sub-local group, 119
- normalising neighbourhood, 119
- notation, xii
- NSS, 106
- NSS approximate group, 208
- one-parameter subgroup, 41, 57
- open mapping theorem, 56
- orthogonal group, 9
- outer regularity, 77
- overspill principle, 155
- periodic group, 233
- Peter-Weyl theorem, 298
- Plancherel identity, 303
- Poincaré-Birkhoff-Witt theorem, 250
- pointed Gromov-Hausdorff convergence, 174
- pointed metric space, 174
- probability space, 313
- profinite group, 130, 134
- projective limit, 93
- pseudometric, 100
- pushforward, 32
- quotient space, 120
- quotiented commutator map, 242
- radially homogeneous $C^{1,1}$ local group, 39
- radical, 255
- Radon measure, 77
- regular representation, 74, 88
- restricted Burnside problem, 235
- restriction of a local group, 27
- Riesz lemma, 64
- Riesz representation theorem, 78
- Ruzsa covering lemma, 183
- Ruzsa triangle inequality, 221
- Sanders lemma, 168, 212
- Schur's lemma, 298
- Schur's theorem, 233
- smooth action, 87
- smooth atlas, 8
- smooth manifold, 8
- smooth structure, 8
- soft analysis, 139
- solenoid, 12
- special orthogonal group, 9
- special unitary group, 9
- spectral theorem, 90
- stabiliser subgroup, 86
- standard function, 152
- standard object, 148
- standard part, 159
- standard universe, 148
- Stone-Cech compactification, 146
- strictly nonstandard, 150
- sub-local group, 119
- subgroup trapping, 112, 122
- submanifold, 54
- syndetic set, 95
- tangent bundle, 31
- tangent space, 30
- tangent vector, 31
- Tonelli theorem, 319
- topological group, 10
- topological manifold, 8
- topological vector space, 61
- torsion group, 162
- totally disconnected space, 128
- transitive action, 86, 127
- triangle removal lemma, 315, 321
- ultra approximate group, 162, 212
- ultrafilter, 145
- ultralimit, 149
- ultrapower, 149
- ultraproduct, 149
- uniform total boundedness, 172
- uniqueness of Lie structure, 42
- unitary group, 9
- universal cover, 226
- universal enveloping algebra, 249
- van Dantzig's theorem, 128
- vector field, 32
- von Neumann's theorem, 54

weak Gleason metric, 104
Weil-Heisenberg group, 9
well-defined word, 30
well-formed formula, 156
word metric, 20