Preface to the third edition

While retaining the same structure and the same expository options, we have extensively expanded the contents of this book for its third edition. This meets a three-fold goal: to take recent advances into account, to flesh out the methodological aspect of the exposition, and to provide basic knowledge or useful supplements for university graduate students, in particular for those preparing for higher teaching diplomas.

Updating with the results from the literature is mostly done in the Notes or Exercises. However, such updates may also be done in new subsections, such as § III.6.5 on Kubilius' model. New proofs of previously included statements are also offered, such as for Tauber's theorem (§ II.7.2) or Halász's (§ III.4.3). Finally, as in the case of the Turán–Kubilius inequality and its friable generalization, the influence of recent results led us to substantially modify the exposition.

Numerous new developments have been inserted in order to preserve general consistency. This essentially concerns: section I.4.7, which is devoted to Selberg's sieve in a little known general form; some applications to small gaps between prime numbers given in the Exercises of the same chapter; the description of Ramanujan's method for the maximal order of the divisor function (Exercise 90); the statements of the Kusmin–Landau inequality (I.6.6) and of van der Corput's general theorem (I.6.10); the inclusion of the explicit formulae of the theory of numbers (§§ II.4.4 and II.8.6); a significant expansion of Chapter II.8, devoted to the distribution of prime numbers in arithmetic progressions; the introduction of Jacobsthal's function and of the proof of Rankin's theorem on large gaps between consecutive primes (§ III.5.6).

Aside from the inclusion in the Exercises of statements following straightforwardly from the main theorems and of synthetic problems, the new items intended for students and future graduate students concern: the Euler–Maclaurin formula (see the exercises of Chapter I.0); an elementary exposition of the Legendre symbol and the theory of quadratic residues (exercises in Chapter I.1); an introduction to the theory of equidistribution modulo 1 (§ I.6.5); a first treatment of Diophantine approximation and a synthetic exposition of continued fractions (Chapter I.7); as well as a *vade mecum* on the theory of Euler's Gamma function (Chapter II.0).

The description sketched above is obviously too succinct to reflect the numerous correlations between developments arising from various motivations. It is also fails to be exhaustive. The text as a whole has been revised, and whole passages have been rewritten. The presentation is further supported by the addition of one hundred and twenty-five new exercises offering, for some important theorems, variations of proofs, or simplified versions, as in the cases of van der Corput's theorem or of the Erdős–Turán inequality. The initial choices of presentation, however, have not been fundamentally modified.

The author wishes to warmly thank all those who have contributed to an attentive and critical rereading of this almost new manuscript, in particular Joseph Basquin, Régis de la Bretèche, Farrell Brumley, Cécile Dartyge, Kevin Ford, Bruno Martin, Michel Mendès France, Aziz Raouj, Jean-Luc Rémy, Olivier Robert, Anne de Roton, Patrick Sargos, and Jie Wu.

Nancy, *November 2007* G.T.

Preface to the English translation

This translation essentially follows the text of the French edition published in 2008, with many corrections and a few updates. It is a pleasure to express here warm thanks to Edward Dunne for his indestructible commitment to making this book available in English, to Patrick Ion, for his careful translation, and to Nicholas Bingham and Matthew de Courcy-Ireland for their invaluable help.

Nancy, *October 2013* G.T.