
Preface

Students tell me that they learn mathematics primarily from doing problems. They say that a good course is one that motivates the material discussed, building on basic concepts and ideas leading to abstract generality, one that presents the “big picture” rather than isolated theorems and results. And, they say that problems are the most important part of the learning process, because the problems force them to truly understand the definitions, comb through the proofs and theorems, and think at length about the mathematics.

Exercises that require basic application of the theorems highlight the power of the theorems. They also offer an opportunity to encourage students to construct examples for themselves. Problems can also be used to explore counterexamples to conjectures. Supplying a counterexample helps the student gain insight into theorems, including an understanding of the necessity of the assumptions. Well-crafted problems review and expand on the material and give students a chance to participate in the mathematical process. Open-ended problems (“Discuss the validity of . . .”) afford the students the opportunity to adjust to researching and discovering mathematics for themselves.

The purpose of this book is to complement the existing literature in introductory real and functional analysis at the graduate level with a variety of conceptual problems, ranging from readily accessible to thought provoking, mixing the practical and the theoretical. Students can expect the solutions to be written in a direct language, one they can understand; always the most “natural” rather than the most elegant solution is presented.

The book consists of twenty chapters: Chapters 1 through 10 contain the Problems, and Chapters 11 to 20 contain (selected) Solutions. Chapters

1 to 7 cover topics in real analysis, from set theory and metric spaces to Fubini's Theorem, and Chapters 8 to 10 cover topics in functional analysis, from functionals and linear operators on normed linear spaces to Hilbert spaces. Each of the Problem chapters opens with a brief reader's guide stating the needed definitions and basic results in the area and follows with a short description of the problems. There are 1,457 problems.

The notation used throughout the book is standard or else is explained as it is introduced. "Problem 2" means that the result alluded to appears as the second item of the chapter in question, and "Problem 3.2" means that it appears as the second item in Chapter 3.

It is always a pleasure to acknowledge the contributions of those who make a project of this nature possible. Mary Letourneau was the best editor and Arlene O'Sean the best project manager this ambitious project could have had. My largest debt is to the students who attended the real variables courses I taught through the years and kept a keen interest in learning throughout the ordeal. Many examples, counterexamples, problems, and solutions are due to them. They also proofread the text and made valuable suggestions. I owe them much. I assume full responsibility for any typos that the text may have and apologize for any confusion they may cause. To quote from John Dryden: "Errors, like straws, upon the surface flow; He who would search for pearls must dive below."

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