Preface

This book is a largely updated write-up of a course on modular forms that the first author gave at several universities between 1976 and 1986.

Some of the sections, in particular the sections on vector-valued modular forms, Weil representations, and Maass forms, are based on lectures given on these topics by the second author, as well as recent results.

It is an *extremely classical* presentation, and we have followed some parts from many excellent books on the subject, such as [**Lan95**] and [**Ser73**]. Other classical books include [**Apo90**], [**DS05**], [**Hid93**], [**Iwa97**], [**Kil15**], [**Kna92**], [**Kno70**], [**Kob93**], [**Miy89**], [**Ogg69a**], [**Ono04**], [**Ran77**], [**Sch74**], and [**Shi94**].

In addition to the standard material which can be found in most of the books cited above, the authors have also strived to include many recent results in order to present a more complete picture of the current state-of-the-art in the theory of modular forms and related topics. This includes both results which are not published elsewhere as well as results from recent publications by the authors and other researchers in the area.

The only prerequisites are some familiarity with ordinary complex analysis and of course a taste for explicit number theory: indeed, the theory of modular forms, together with its close cousin the theory of elliptic functions, which we will also mention, is probably one of the theories which contain the largest number of explicit identities.

Indeed, the present book contains a *huge* number of identities, and it is unavoidable that some errors remain (misprints or otherwise) and the authors would greatly appreciate receiving corrections.

In most of the book, we mainly consider functions which are modular for the full modular group $\mathrm{PSL}_2(\mathbb{Z}) = \mathrm{SL}_2(\mathbb{Z})/\{\pm I\}$ or its subgroups of finite index and in particular on its standard congruence subgroups. In Chapter 15, we will give an introduction to more general types of modular forms, including brief expositions of half-integral weight, Jacobi, Maass, Hilbert, and Bianchi modular forms.

We expect the main use of the book to be for advanced graduate students to learn about the classical theory of modular forms. However, we also believe that the book will be useful as a reference for active researchers in the area due to the multitude of explicit formulas and identities.

To get the most out of the book, the interested reader should pay attention to the methods used in the proofs and should work through all the exercises as well as try to implement the formulas and algorithms in a computer algebra package of his or her choice.

Note that many proofs are quite computational in nature, and following the proof in detail may sometimes be rather tedious. However, we emphasize that we will *never* use any deep mathematical tools (no deeper than standard complex analysis and number theory), and in particular we will not discuss the much more abstract theory of *automorphic representations* or *automorphic forms on Adele groups*. We will also not discuss the theory of mod p or p-adic modular forms.

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