
Preface

The idea of writing this book came from the collaboration of the authors. We started working on quiver representation when the first author visited Northeastern University as a Research Scholar in the academic year 1997/1998. We both came from other fields of algebra so we had to learn the subject from scratch. At the same time we were interested in connections of quiver representations with invariant theory, representations of algebraic groups, and algebraic geometry.

This experience made us realize that (except for Crawley-Boevey's notes [18] available online), there is no introductory text which would allow a person without any knowledge of Artin algebras to learn the subject quickly.

Later both authors taught courses on quiver representations at University of Michigan and Northeastern University, respectively. The second author also taught a two month course on quiver representations at Tor Vergata University in Rome in the spring of 2007.

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The book is addressed to non-specialists, who want to learn the subject without going through the extended preparation in algebra, just starting from basic linear algebra. It turned out to be impossible to be completely elementary, so in some places we use some basic algebraic geometry (mainly the dimension counts).

We do not prove the results in full generality, working over the field \mathbb{C} of complex numbers. We also work mostly with acyclic quivers. We only cover Auslander-Reiten duality in the case of hereditary algebras.

The book reflects our point of view, so the semi-invariants are covered in detail and we stress their role in our approach. Some of the results could be proved just using stability conditions without mentioning semi-invariants, but we find the combinatorics of the rings of semi-invariants quite fascinating.

Still many important topics are left out, for example, Ringel-Hall algebras and Nakajima quiver varieties.

In recent years the field developed very quickly. New concepts and connections emerged. We wanted this development to be reflected in the book, hence there are chapters on orthogonal categories, exceptional sequences and cluster categories.

We stress the connections of quiver representations with representations of algebraic groups and moduli problems.

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