
Index

- abelian scheme, **147**, 148, 186
abelian variety, 92, 115, **142–150**,
 151–153, 158, 162, 163, 207, 216,
 225, 226, 229, 245, 247, 276, 290,
 292, 310
adèle ring, **3**, 50, 53, 59, 98, 155, 231
adjoint algebraic group, **139–140**, 141,
 154, 310
affine algebraic group, **130–131**, 151,
 153
affine morphism, **104**
Albanese torsor, **158**, 166
Albanese variety, **145**, 146, 147, **158**,
 158, 166
algebraic cycle, **222–223**
algebraic group, **125**, 119–168
 quasi-projectivity of, 129
algebraic space, 39, 90, 186
almost simple algebraic group,
 140–141, 141, 142, 155
 α_p , **124**, 132, 164, 166, 207
 α -twisted sheaf, 195
Amitsur’s conjecture, **113**
anti-affine algebraic group, **153**
Azumaya algebra, **16**, 16–30, 111, 113,
 191, **193**, 193–195, 207, 238, 240
- base extension, **32–37**, 41, 48, 101, 105,
 116, 126, 303–311
Betti number, 209, 210, 213, **215–216**,
 221
bielliptic surface, **253**
birational automorphism group, **94**, 269
birational invariant, 95, 202, 261, 262,
 268, 270, 281, 282, 293
birational map, 94, 113, 117, 202, 243,
 262, 263, 265, 274
 factorization of, 272
birational morphism, 192, 271–275, 280
 factorization of, xiii, 272
Birch and Swinnerton-Dyer conjecture,
 115, **150**, 225
Bombieri–Lang conjecture, 292
Borel–Serre theorem
 global, **163**, 188
 local, **162**, 189, 245
Brauer evaluation, **233**, 235–236, 238,
 258
Brauer group, 1, **19**, 16–27, 111–114,
 117, 170, 171, **191**, **193**, 190–196
 algebraic part of, **199**, 205
 Azumaya, **193**, 193–196, 203
 birational invariance of, 202
 cohomological, **191**, 191–193
 counterexamples, 192, 196, 201–202,
 207
 of a C_1 field, **24**, 201
 of a complete intersection, 257
 of a curve, 204, 207, 208
 of a del Pezzo surface, 258, 282
 of a discrete valuation ring, 200–201,
 203, 208, 233
 of a finite field, **25**, 167, 203
 of a function field, 25, 27, 30, 202
 of a global field, 21, **26**, 112
 of a Laurent series field, 201

- of a limit, 29, 179, 192
 of a local field, 21, **25**
 of a maximal abelian extension, **30**
 of a perfect field, 30
 of a projective space, **204**
 of a proper \mathbb{Z} -scheme, 204
 of a quadric, **205**, 208
 of a quadric bundle, **205–206**
 of a rational variety, **204–205**
 of a regular integral scheme, 192,
 201–202
 of a ring of S -integers, **203–204**, 208
 of a singular variety, 196, 207
 of a surface over a finite field, 204,
 225, 226
 of an adèle ring, 233
 spreading out, **193**, 233
 Brauer set, 232, **234**, 251
 Brauer–Manin obstruction, **234**,
 233–240, 250, 258, 282, 289
 effectivity of, 240
 for a rationally connected variety, 271
 insufficiency of, 253–257
 to weak approximation, **236**, 240,
 258, 289
- C_1 field, 3, 6–8, 24, 25, 27, 28, 201
 C_r field, 1, **3**, 3–8
 counterexamples, 7, 24, 28
 examples, 6–7
 for non-integral r , 7, 27
 transition theorems for, 5–6, 28
- C'_r field, **7**, 28
 canonical ring, **262**
 Čech cohomology, 161, 163, 177,
181–182, 184–186, 190, 191, 244
 center
 of a group scheme, **128**
 of a reductive group, 138
 of a semisimple group, 138
 central isogeny, **138**, 139, 140
 centralizer of a subgroup scheme, **128**
 Chabauty’s method, 243
 character group, **134**, 135, 136, 166
 Châtelet surface, **237**, 239, 254
 Chevalley–Warning theorem, **6**, 25, 27
 Chow group, **222**, 224
 class field theory, 27, 200, 299
 classification of surfaces, 271–279
 closed point, **45**, 45–47, 54, 63, 212
 cocharacter group, **165**
 Cohen structure theorem, 87
 cohomological dimension, 1, **13**, 12–16,
 24, 29, 30
 examples, 15–16
 of a global field, 15, 200
 of a local field, 15
 strict, **13**, 13–15, 200
 transition theorems for, 13–15
 cohomology with compact support, 221,
 223, 227
 commutator subgroup of an algebraic
 group, **136**, 142, 153
 comparison of Azumaya and
 cohomological Brauer groups, 191,
194–196
 comparison of Brauer–Manin and
 descent obstructions, 250–251
 comparison of étale and fppf
 cohomologies, 190–192
 comparison of étale and Galois
 cohomologies, 11, 169–170, 180
 comparison of étale and singular
 cohomologies, 170–171, 214, 221
 comparison of étale and topological
 fundamental groups, 80
 comparison of étale and Zariski
 cohomologies, 190, 207
 comparison of étale–Brauer and descent
 obstructions, 251
 complete intersection, **257**, 288
 complex analytic space, 50, 70, 76, 80
 complex manifold, 70, 170
 component group of an algebraic group,
151, 246
 composition series, **127**, 132
 conic, 208, **237**, 280, 281, 283, 284, 288
 conic bundle, xiii, **237**, 240, 278, 279
 connected component of a group
 scheme, **125**, 129, 151, 152, 187
 constant field
 of a (-1) curve, **271**
 of a function field, **37**
 constructible scheme, 68
 contracted product, **160**, **187**
 contractible curve on a surface, **271**,
 273, 274
 Cremona group, **94**
 curve, **32**, 47–49, **271**
 cohomology of a, 170, 207, 210
 of genus 0, 110, 117, 285
 of genus 1, 158, 161, 242, 258, 259
 of genus > 1 , 51, 290–292

- cycle class homomorphism, **224**, 222–226
- cyclic algebra, **22**, 22–23, 25–27, 29, 30, 194
- cyclotomic character, 217
- Dedekind domain, **64**, 64–66, 90, 91, 98
- Dedekind scheme, **64**, 64–66, 91, 92, 148, 275
- degree of a closed point, **46**
- degree of a Fano variety, **268**, 280
- degree of a morphism, **94**, 96
- del Pezzo surface, **280**, 278–290
 - of degree 1, **282**, **289**
 - of degree 2, **282**, **289**
 - of degree 3, **289**
 - of degree 4, 258, 288, **288**
 - of degree 5, xiii, 286, **286–288**, 294
 - of degree 6, xiii, **285–286**
 - of degree 7, **284–285**, 286
 - of degree 8, **282–284**, 286
 - of degree 9, **282**, 285
- Demyanenko–Manin method, 243
- descent
 - fpqc, 101–117, 126, 127, 175
 - for algebraic groups, 129, 308
 - for properties of morphisms, 105, 183, 189, 199, 207, 303, 304, 306, 307
 - for quasi-coherent sheaves, **104**, 176, 190, 194, 223
 - for schemes, **105**, 176, 185–187, 223
 - Galois, 9–11, 20, 54, 106–109, 137, 185
 - generalizations of Fermat’s method of infinite, 231, 240–250, 258
 - descent datum, **103**
 - descent obstruction, **249**, 248–252
 - insufficiency of, 254, 257
 - iterated, **252**
 - dimension, **67**
 - dimension ≤ 1 , **24**, 24, 30, 160, 281, 282, 284–286
 - diophantine approximation, 291
 - discrete valuation ring or field, 2, 7, 15, 62–63, 65, 66, 75, 90–92, 116, 147–149, 200–201, 203, 208, 275, 299–301
 - dominant rational map, 43, 54, **93**, 93–94, 248, 263–267, 271, 287, 306
 - elliptic curve, 117, 123, 142, 147–150, 154, 158, 241, 242, 245, 254
 - elliptic surface, **277**
 - embedding of algebraic groups, **128**
 - Enriques surface, **277**
 - étale algebra, **8**, 8–9, 22–23
 - split, **9**, 10, 283
 - étale cohomology, 11, **178**, 169–208, 214, 216–230, 264
 - and limits, 179
 - of a curve, 170, 207
 - over a field, 180
 - étale morphism, 8, **76**, 76–80, **85**, 96, 97, 99, 172, 243, 247
 - étale schemes over a normal scheme, 82–84, 248
 - étale-Brauer obstruction, 253
 - insufficiency of, 253–256
 - étale-Brauer set, **251**, 251
 - Euler characteristic, 213, 220, 277
 - of a sheaf, **293**
 - exact sequence of group schemes, **124**
 - exact sequence of sets, **174**
 - exact sequence of sheaves, **178**
 - extension of group schemes, **124**, 308
 - F*-obstruction, **231–233**
 - faithfully flat morphism, **67**, 68, 306
 - Fano variety, **268**, 268–270
 - Fermat’s last theorem, 54, 296
 - fiber functor, **78**, **80**
 - fibration method, 240
 - field of moduli, 116
 - finite field, xiii, xiii, 6, 15, 25, 27, 49, 54, 160, 161, 167, 208–230, 279, 294
 - finite presentation morphism, **59**, 59–62, 306
 - finitely presented algebra, **58–59**
 - finitely presented group, 81
 - flat module, **66**, 66
 - flat morphism, **67**, 306
 - formally real field, 7, 15, **299–300**
 - formally smooth morphism, **85**, 306
 - fppf cohomology, 21, 111, 161, 163, **178**, 204
 - fppf morphism, **68**, 97, 99, 124, 172, 303, 306
 - fpqc morphism, **69**, 97, 99, 103, 172, 303–306
 - that is not fppf, 99
 - free rational curve, **265**

- Frobenius automorphism, 150, 161, **218–219**
 Frobenius morphism, **95–97**, 214, 227, 291
 of a group scheme, 123, 124, 128, 129, 138, 157, 161
 function field of a variety, **34**, 34–37, 93–94, 291–292
 functor of points, **38**, 38–39, 59, 91, 92
 is a sheaf, 176
 of a group scheme, **121–122**, 123, 126, 143, 144
 fundamental group, **79**, **80**, 78–81, 257
 of a normal scheme, 83
 variety with prescribed, 81
 G-unramified morphism, **76**, 76, 85, 307
 GAGA, **49**
 Galois cohomology, 11–12, 19–23, 28, 169–170, 180
 Galois étale algebra, **9**
 Galois theory, 8–12
 gamma function, **211**
 general type, **263**, 277
 genus, **47–48**
 change under field extension, 48
 geometric point, **80**
 geometrically connected variety, **33**, 34, 44, 53, 87, 125, 143, 152, 161, 306
 geometrically integral variety, 2, **33**, 34–37, 40, 44, 45, 47, 48, 54, 74, 83, 84, 87, 98, 116, 143, 145, 157, 166, 167, 199, 205, 206, 208, 226, 227, 236, 247, 248, 255, 256, 262, 267, 291, 292, 302, 304, 306
 geometrically irreducible variety, **33**, 34–36, 44, 83, 125, 210, 213, 226, 227, 229, 306
 geometrically reduced variety, **33**, 34–36, 44, 45, 53, 86–88, 128, 145, 273, 274, 278, 304, 306
 geometrically regular variety, **33**, 34, 36, 53, 71, 74, 75, 87
 global field, xiii, **2–3**, 26–27, 50–53, 55, 149–150, 154–155, 163–164, 226–227, 231–259, 281–292
 global function field, **2**, 51, 155, 163, 164, 225, 226, 291–292
 good reduction, **90**, 90–91, 98, 147, 148
 Grassmannian, **263**, 287
 Greenberg transform, **116**, 116
 Grothendieck topology, **171**, 171–173
 group object, **120**
 group scheme, 11, 92, **120**, 119–125, 177
 additive, **122**, 123, 124, 131–133, 155, 164–166, 177, 308
 constant, **123**, 132, 133, 157
 finite étale, 150–152
 fppf, **124**, 124–129
 multiplicative, **122**, 122–124, 134, 136–138, 148, 164–166, 177, 184, 190–191, 207
 of multiplicative type, **136**
 of order p , 164
 over the ring of dual numbers, 131
 Grunwald–Wang theorem, 26
 Hasse–Weil bound, 162, **210**, 258
 height of a rational point, **51**, 149
 Hensel's lemma, **85**, 86, 188, 227, 239, 248, 300
 henselian local ring or field, 7, 203, 235, **300–301**
 Hermite's finiteness theorem, xiii, **189**
 Hilbert irreducibility theorem, **301**
 Hilbert's tenth problem, **50**, 50
 Hilbert's theorem 90, **11–12**, 21, 28, 191, 200
 hilbertian field, **301–302**, 302
 Hirzebruch surface, **264**, 276, 279
 Hodge conjecture, 224, 225, 270
 Hodge structure, **165**
 homogeneous space, xiii, 126, 129, **157**, 167
 Hopf algebra, 121
 hyperbolic variety, **257**
 hyperelliptic or superelliptic curve, 116
 hypersurface, 6, 205, 214, 257, 268, 270, 293
 imperfect field, 27, 34, 48, 69, 74, 86, 88, 117, 200–202, 207
 group scheme over an, 153–154, 164
 index of an Azumaya algebra, **21–22**, 25, 26
 induced module, **13**
 infinitesimal lifting property, **85**, 84–86
 inflation-restriction sequence, 21, 189, **198**
 inner twist, **155**, 159, 163, 186
 integral scheme, **31**
 intermediate Jacobian, 269
 isogeny between connected algebraic groups, **138**

- isotrivial variety, **292**
Jacobian variety, **143**, 143–145, 158, 162, 226, 243
jet, **85**
jet space, **116**
K3 surface, 51, 90, **276**
Kodaira dimension, **261**, 261–263, 276–277
Krasner’s lemma, 88, **89**, 245, 248
Kronecker–Weber theorem, **7**
L-function, **150**, 150, 225
Lang’s conjecture for a hyperbolic variety, 257
Lang–Nishimura theorem, xiii, **94–95**, 99, 111, 285, 287
Lang–Weil theorem, **226**, 229
Laurent series, **xiv**, 1, 2, 7, 30, 50, 94, 95, 201, 299
Lefschetz trace formula
 for a nonproper variety, **221**, 226
 in étale cohomology, **218**, 220, 279
 in topology, 214, **215**
linear recursive sequence, **230**
local coordinates, 71, **81**, 87
local field, **1–2**, 7, 15, 21, 25, 27, 49, 50, 88–89, 162–163, 203, 228, 235, 245, 301, 302
 higher-dimensional, **299**
local-global principle, **52**, 231, 232, 234, 236, 239, 240, 249, 253, 254, 284
 for a Châtelet surface, 240
 for a del Pezzo surface, 282, 284, 288–290
 for a hypersurface, 257
 for a rationally connected variety, 271
 for a Severi–Brauer variety, 112, 282, 285
 for a torsor, 163–164
locally integral scheme, **64**
locally of finite presentation morphism, **59**, 68, 306
maximal abelian extension, 7, 30
maximal unramified extension, 7, 83, 201, 301
Merkurjev–Suslin theorem, **24**
Minchev’s theorem on strong approximation, xii, **247–248**
minimal regular proper model, 91, 92, 148, 149, **275**
minimal surface, **274**, 274–275, 278
moduli space, **39–41**, 263
 coarse, **39**, 40, 41, 54, 277
 fine, **39**, 40, 54
 of curves, **40**
 of curves with marked points, **40–41**, 286, 287
Mordell–Weil theorem, **149**, 247
 μ_n , **123**, 124, 125, 133, 136, 138–141, 164, 166, 194, 195, 207, 217, 223
nef, **276**, 276, 278
Néron model, **91–92**
 of an abelian variety, 147–148
 of an elliptic curve, 148–149
nice variety, **87**
noetherian hypothesis, 58, 61–62, 304
norm form, **3–4**, 24, 28
normalizer of a subgroup scheme, **128**
normic form, **3–5**
number field, **2**, 7, 28, 51, 52, 154, 155, 163, 164, 188, 189, 200, 227, 239, 245, 247, 251, 257, 259, 271, 290–292
numerical equivalence, **276**, 278
opposite category, **xiv**
order of a group scheme, **123**
PAC field, 8, **302**
perfect closure, **1**, 27
period of an Azumaya algebra, **21–22**, 25, 26
Picard number, **278**
Picard scheme, **144**, 146
Picard variety, **146–147**
Poincaré duality
 for a nonproper variety, **221**, 223
 in ℓ -adic cohomology, **218**
 in topology, **215**
polynomial ring, **xiv**, 58
potential density, **52**
power series ring, **xiv**, 87, 94, 95, 99, 201
presheaf, **173–174**
primary extension of fields, **36**, 37, 44, 206
pseudo-morphism, 93
pseudo-reductive algebraic group, **153**, 162, 166
pseudo-semisimple algebraic group, **153**
purity conjecture, 202
quadric, **205**

- quadric bundle, **205–206**, 253–256
quasi-affine morphism, **105**, 105, 109,
307
quasi-compact morphism or scheme,
57–58, 307
quasi-separated morphism or scheme,
57–58, 307
quaternion algebra, **17**, 18, 22, 23, 29,
30, 238, 258
quotient of group schemes, **124–125**,
126–128, 308
radical of an algebraic group, **136–137**,
151, 153
rational equivalence, 222, 224
rational function field, **xiv**, 5, 27, 28,
30, 117
rational map, **92–95**, 98, 99, 112–114,
147, 204, 261, 263, 265, 266, 285
rational surface, 264, 276, 278, 279, 281,
284–286, 288, 290
cohomology of a, 264
number of points on a, 279
rational variety, 48, 70, 111, 135, 165,
204–205, **263**, 263–264, 268–270,
270
in a family, 264
rationally chain connected variety, 6,
267–270, 279
rationally connected variety, 264,
266–267, 268–270, **270**
real approximation theorem, **154**, 166
real closed field, **300**
reduced norm, **18**, 24, 29
reduced subscheme of a group scheme
that is not a subgroup scheme, 166
reduced trace, **18**, 29
reductive algebraic group, **137**,
136–142, 152, 162, 310
regular scheme, 34–36, 48, 53, 64, 69,
69–70, 73–75, 81, 91, 93, 98, 148,
149, 192, 196, 201–202, 204–206,
251, 271–275, 308
relative dimension, **67**, 72–73, 77, 81
relative Picard functor, **144–145**
representable functor, **39**, 39, 40, 50,
54, 114, 124, 128, 144, 145, 165,
185–187
residue homomorphism, 196, **200–202**,
206, 208, 238
resolution of singularities, 35, 91, 262
restricted product, **xiii**, 3, 50, 53
restriction of scalars, **114–116**, 117,
134, 141, 142, 165, 166, 278, 283,
284, 288
inseparable, 117
Riemann existence theorem, **80**, 80
Riemann hypothesis, **211**
for a curve over a finite field, 214
for a variety over a finite field, **210**,
213, 214
Riemann–Roch theorem, 47, **48–49**, 54,
162, 263, 293
ring of (*S*-)integers, **2**, 64, 83, 203–204,
207, 208, 212, 258
ruled surface, 276, 278, 279
ruled variety, **265**, **270**
scheme-theoretically dominant, **43**, 54
scheme-valued points, **38**, 37–45
Schinzel’s hypothesis, 240
Selmer group, 245
Selmer set, **245–246**, 249
finiteness of, xii, 187, **246**, 247
semiabelian variety, **143**, 145, 148, 158
semisimple algebraic group, **137**,
136–142, 151, 152, 154, 155, 162,
164, 310
separable extension of fields, 7, **35–36**,
37, 53
set theory, xiv, 17, 171, 177, 295–298
Severi–Brauer variety, **110**, 110–114,
117, 208, 240, 282, 284, 285
Shafarevich conjecture, **290**
Shafarevich–Tate group, **149–150**,
225–226
Shapiro’s lemma, **13**, 14, 205, 284
sheaf, **175**, 174–178
sheafification, 126, 144, **177**, 178
sieve, 172
simply connected algebraic group, xiii,
139, 139–142, 154, 155, 162, 164,
309, 310
simply connected scheme
algebraically, **80**, 83, 139, 248, 256,
257, 277
simply connected topological space, **79**,
80, 266
singular cohomology, 170, 210, 214–216,
221, 224
site, **172–173**
Skolem–Mahler–Lech theorem, **230**
Skolem–Noether theorem, **18**
small category, **297**

- smooth group scheme, 92, 125,
128–129, 149
 smooth locus, **73**, 86, 92, 129, 149, 227,
 248
 smooth morphism, **72**, **75**, **84**, 69–92,
 307
 smooth vs. regular, 69, 73–75
 solvable algebraic group, **137**, 137, 151,
 310
 specialization, **62**, 132, 221, 301
 spectral sequence, **196–200**
 Hochschild–Serre, 21, 196, **198–200**
 of Čech cohomology, **182**
 spreading out, 59, **60–66**, 193, 226, 227,
 233, 243, 246, 247, 258, 303–307
 stably birational varieties, **114**, 117
 stably rational variety, **263–264**,
 268–270
 stack, 39, 40
 standard étale morphism, **77**, 78, 88
 strictly henselian local ring or field, **301**
 strong approximation, xii, **53**, 99,
 247–248
 in an algebraic group, **155**
 strongly inaccessible cardinal, **296**
- Tate class, **225**
 Tate conjecture, **224**, 224–226
 Tate module, 150, 216, 229
 Tate twist, **217**, 218
 topological field, 2, 49–50
 torsor, 122, **156**, 156–164, 166, 167,
 169, 171, **183**, 182–189, 194, 199,
 207, 231, 244–259
 operations on a, 159–160, 186–187
 over a finite field, 160–162, 188
 over a local field, 162
 under a torus, 157, 167, 286
 under α_p , 207
 under μ_n , 194, 207
 torsor sheaf, **184**, 184–186
 torus, **134**, 134–138, 142, 143, 151, 152,
 155, 157, 164–167, 286, 310
 rationality of a, 135
 transcendence degree, 15
 transcendence basis, **xiv**
 transcendence degree, **xiv**, 5, 16, 35, 67,
 262, 292
 Tsen’s theorem, **6**, **25**, 204
 twist, 16, 20, 41, **109**, 109–116, 132,
 134, 135, 141, 142, 155, 158–160,
- 185, 187, 242, 244, 251, 253, 256,
 258, 283, 284, 287
 of an elliptic curve, 117
 of \mathbb{G}_a , 131, 164–165
- unipotent group, **132**, 131–133, 137,
 152, 166, 310
 unipotent radical, **136–137**, 151, 153
 unirational variety, **263–264**, **270**, 282,
 288, 290
 that is not rational, 268–270
 uniruled variety, **265**, 268–270, **270**
 universe, xiv, 17, 177, **295**, 295–298
 unramified cohomology class, **187**
 unramified cohomology group of a
 function field, 270
 unramified morphism, **75–76**, 77, 82,
 83, 307
 unramified torsor, 187–189
- valuative criterion for properness,
65–66, 91, 93, 95, 188, 246
- variety, **32**
 vectorial group, **131**
 very free rational curve, **265**
- weak approximation, **52**, 55, 99, 232,
 236, 240, 258, 282, 288–290
 in an algebraic group, **154–155**
 weak Lefschetz theorem, **257**
 weak Mordell–Weil theorem, 149, **247**
 Wedderburn’s theorem on finite division
 rings, 25, 167
 Wedderburn’s theorem on semisimple
 algebras, 17
 Weierstrass model, 117, 148, **148–149**
 Weil conjectures, **209**, **213**, 209–230
 Witt vector, 116, 132, 133
- Yoneda’s lemma, **39**, 91, 92, 122, 131
- Zariski open covering morphism, 69, **97**,
 97, 99, 103, 304, 305
 Zariski’s main theorem, 78, 82
 zeta function, 211–214
 convergence of, 212, 229
 Dedekind, **212**
 of a scheme of finite type over a finite
 field, 212–213
 of a scheme of finite type over \mathbb{Z} , **212**
 Riemann, **211–212**, 212