
Index

- $(\mathcal{L}_1 \cdots \mathcal{L}_t)$, 369
- $(\mathcal{L}_1 \cdots \mathcal{L}_t; \mathcal{F})_V$, 368
- $(\mathcal{L}_1 \cdots \mathcal{L}_t \cdot W)$, 369
- (ϕ) , 286
- $(\phi)_X$, 286
- (f) , 241
- $(f)_0$, 241
- $(f)_\infty$, 242
- A-torsion, 208
- $B(I)$, 113, 121
- $B(\mathcal{I})$, 221–223
- $C^*(\underline{U}, \mathcal{F})$, 310
- $D(B/A)$, 399
- $D(F)$, 71, 103
- $D(f)$, 48
- $D \cap U$, 242
- $D \equiv 0$, 382
- $D_1 \geq D_2$, 240
- $D_1 \sim D_2$, 242
- $D_{L/K}(a_1, \dots, a_n)$, 398
- $F : X_p \rightarrow X$, 349
- F^a , 71
- $Fr(x)$, 349
- $G(L, K)$, 7
- $G(Y/X)$, 421
- $G^i(S/R)$, 440
- $G^s(S/R)$, 438
- $H^i(A^*)$, 307
- $H^i(X, F)$, 308, 309
- $H_Y^i(X, \mathcal{F})$, 330
- $H_1^i(M)$, 326
- $H_{\text{Sing}}^0(U, G)$, 358
- $I(S/R)$, 435
- $I(Y)$, 29, 33, 69, 70, 103
- $I(Z)$, 292
- I^{sat} , 66
- $I_X(Y)$, 33, 70
- $J(S/R)$, 435
- K^s , 438
- K_X , 286
- $M(n)$, 63
- M^\vee , 243
- $M_{(F)}$, 64
- $M_{(\mathfrak{p})}$, 64
- $N(X)$, 382
- $N_{L/K}$, 392
- $P(A)$, 243
- PG_X , 189
- $P_M(z)$, 301
- P_Y , 301
- $R^i \phi_* \mathcal{F}$, 320
- $R^{(d)}$, 64
- $S(I; R)$, 56
- $S(W)$, 103
- $S(X)$, 70
- $S(X \times \mathbb{P}^n)$, 105
- $S(Y)$, 105
- $S(\mathbb{P}^m \times \mathbb{P}^n)$, 102
- $S(\mathbb{P}^n)$, 68
- $T(A)$, 24
- $T_A(M)$, 208
- $T_p(X)$, 157
- V_ν , 228
- W_F , 103
- $X \times Y$, 101
- $X^{(r)}$, 360

- X_F , 71
 X_Z , 290
 X_f , 48
 X_p , 290
 Y^H , 422
 $Z(A)$, 102
 $Z(I)$, 68
 $Z(J)$, 70
 $Z(T)$, 28, 33
 $Z(U)$, 68
 $Z(f)$, 28
 $Z_1 \cap Z_2$, 290
 $Z_X(T)$, 33
 $Z_X(U)$, 70
 Z_{red} , 289
 $[L : K]_i$, 7
 $[L : K]_s$, 7
 $\mathbb{A}^m \times \mathbb{A}^n$, 101
 \mathbb{A}^n , 27
 $\mathbb{C}\{x_1, \dots, x_n\}$, 176
 Δ , 176
 Δ_X , 110
 $\Delta_{\mathbb{P}^n}$, 106
 $\Gamma_Y(X, \mathcal{F})$, 330
 Γ_ν , 228
 Γ_ϕ , 106, 107
 \mathbb{N} , 1
 $\Omega_{X/k}^n$, 286
 $\Omega_{k(X)/k}^n$, 286
 $\Omega_{B/A}$, 280
 $\Omega_{X/k}$, 283
 $\mathbb{P}^m \times \mathbb{P}^n$, 102
 \mathbb{P}_k^n , 67
 \mathbb{Z}_+ , 1
 \mathbb{Z}_{an} , 359
 $\hat{H}^p(X, \mathcal{F})$, 312
 $\hat{H}^p(\underline{U}, \mathcal{F})$, 311
 $\chi(\mathcal{F})$, 319
 $\deg x$, 63
 \deg , 250
 $\deg(D)$, 254
 $\deg(Y)$, 301
 $\deg(\phi)$, 254, 371, 406
 $\dim R$, 19
 $\dim X$, 42, 139
 $\ell_R(M)$, 9
 \hat{M} , 409
 $\hat{\phi} : \hat{R} \rightarrow \hat{S}$, 411
 $\lim_{\leftarrow} A_i$, 184
 $\lim_{\rightarrow} A_i$, 182
 $\mathcal{F}(n)$, 210
 $\mathcal{F} \otimes \mathcal{G}$, 204, 206
 $\mathcal{F} \otimes_{\mathcal{O}_X} \mathcal{G}$, 204, 206
 \mathcal{F}^{an} , 359
 \mathcal{G}^* , 245
 $\mathcal{I}\mathcal{O}_X$, 205
 \mathcal{I}_Y , 155, 198, 200
 \mathcal{I}_Z , 289
 $\mathcal{I}_{Y,p}$, 155
 $\mathcal{I}_{\text{div}(s)}$, 291
 \mathcal{L}^{-1} , 269
 \mathcal{O}_D , 290
 $\mathcal{O}_U(V)$, 53
 \mathcal{O}_X -module, 191
 \mathcal{O}_X -module homomorphism, 191
 \mathcal{O}_X -submodule, 192
 \mathcal{O}_X -torsion, 209
 $\mathcal{O}_X(D)$, 242
 $\mathcal{O}_X(U)$, 49, 75, 187
 $\mathcal{O}_X(n)$, 210
 \mathcal{O}_X^* , 271
 $\mathcal{O}_{U,p}$, 53
 $\mathcal{O}_{W,(p,q)}$, 103
 $\mathcal{O}_W(U)$, 103
 $\mathcal{O}_{X,E}$, 240
 $\mathcal{O}_{X,Y}$, 229
 $\mathcal{O}_{X,p}$, 50, 75
 $\mathcal{T}(\mathcal{F})$, 209
 $\mathfrak{p}_i^{(a_i)}$, 245
 $\text{Ann}_X(\sigma)$, 208
 $\text{Aut}(X)$, 351
 $\text{Base}(L)$, 261
 $\text{Cl}(X)$, 242
 $\text{Cl}^0(X)$, 258
 $\text{Cokernel}(\alpha)$, 194
 $\text{Der}_A(B, M)$, 279
 $\text{Div}(F)$, 249
 $\text{Div}(X)$, 240
 $\text{Grass}(a, b)$, 73
 $\text{Image}(\alpha)$, 194
 $\text{Kernel}(\alpha)$, 194
 $\text{Map}(U, \mathbb{A}^1)$, 49, 75
 $\text{Pic}(X)$, 270
 $\text{Pic}^{\text{an}}(X)$, 359
 $\text{Proj}(S)$, 67, 297
 $\text{Spec}(R)$, 5, 296
 $\text{Supp } D$, 240
 $\text{bideg}(x)$, 102
 $\text{div}(\phi)$, 286
 $\text{div}(\phi)_X$, 286
 $\text{div}(f)$, 242
 $\text{div}(f)_X$, 242

- $\text{div}(s)$, 291
 $\text{gcd}(D_0, \dots, D_n)$, 259
 $\text{ht}(\mathbf{P})$, 19
 $\text{reg}(M)$, 328
 $\text{reg}(\tilde{M})$, 328
 $\text{trdeg}_K L$, 6
 $\mu(I)$, 46
 ν_E , 241
 $\phi : X \dashrightarrow Y$, 58, 93
 ϕ^* , 35, 76
 $\phi^* : \text{Cl}(Y) \rightarrow \text{Cl}(X)$, 252
 $\phi^* \mathcal{M}$, 205
 $\phi^{-1}(Z)$, 290
 ϕ_L , 260
 ϕ_V , 260
 $\text{Vol}(\mathcal{L})$, 345
 \sqrt{I} , 3
 $\sqrt{\mathcal{I}}$, 289
 $\text{Tr}_{L/K}$, 392
 \tilde{I} , 120, 198, 200
 \tilde{J} , 192
 \tilde{M} , 196
 \tilde{N} , 199
 $|D|$, 260
 $\wedge^n M$, 286
 $a^i(M)$, 327
 $d(R' : R)$, 443
 $d\phi_p$, 157
 $e(R)$, 376
 $e(S_i/R)$, 396
 $e(\nu^*/\nu)$, 397
 e_P , 346
 $f(S_i/R)$, 396
 $f(\nu^*/\nu)$, 397
 $f^* \mathcal{G}$, 207
 f^h , 71
 $f^\#$, 191
 $f_* F$, 191
 $g(R' : R)$, 443
 $g(X)$, 334
 $h^i(X, \mathcal{F})$, 319
 k , an algebraically closed field, 1
 k -linear Frobenius map, 349
 $k(U)$, 53, 82
 $k(W)$, 103
 $k(X)$, 49, 74
 $k(p)$, 50, 75, 283
 $k[X]$, 33
 $k[\mathbb{A}^m \times \mathbb{A}^n]$, 101
 $k[\mathbb{A}^n]$, 27
 m -regular, 328
 m_R , the maximal ideal of a local ring, 1
 m_p , 50, 75, 283
 $m_{X,p}$, 50, 75
 n -fold, 45
 p -basis, 425
 p -independent, 425
 $r(R' : R)$, 443
 $\text{Aut}(L/K)$, 7
 $\text{Aut}(k(Y)/k(X))$, 421
 $\text{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{G})$, 201
 $\text{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{G})$, 204, 206
 $\text{Num}(X)$, 382
 $\text{QF}(R)$, 1
 $\text{QR}(A)$, 291
 $\text{codim}_X(Y)$, 45
 $\text{depth}_I M$, 326
 res , 34
 $g^r_{m_R}(R)$, 23
Abel, 360
Abelian variety, 360
Abhyankar, xii, 170, 227, 228, 231, 232, 234, 235, 392, 413, 435
Abhyankar-Jung theorem, 436
Abramovich, 234
Abstract prevariety, 295
Abstract variety, 295
Adjunction, 287, 380
Affine map, 127
Affine scheme, 296
Affine subscheme, 289
Affine subvariety, 34
Affine variety, 55
Algebraic function field, 7
Algebraic local ring, 441
Algebraic set, biprojective, 103
Algebraic set, in \mathbb{A}^n , 28
Algebraic set, projective, 68
Algebraic set, quasi-affine, 34
Algebraic set, quasi-biprojective, 103
Ample divisor, 262
Ample invertible sheaf, 270
Analytic implicit function theorem, 176
Artin, 362
Auslander, 25, 433
Base locus, of a linear system, 261
Base point free linear system, 261
Bayer, 330
Bertini, 304, 451
Bézout, 375
Bézout's theorem, 375

- Bhatt, 433
 Bihomogeneous coordinate ring, 102
 Birational equivalence, of affine varieties, 60
 Birational equivalence, of quasi-projective varieties, 94
 Birational map, of affine varieties, 60
 Birational map, of quasi-projective varieties, 94
 Birkar, 387
 Blow-up, of a subvariety of a projective variety, 123
 Blow-up, of a subvariety of an affine variety, 113
 Blow-up, of an ideal of a projective variety, 121
 Blow-up, of an ideal of an affine variety, 113
 Blow-up, of an ideal sheaf on a projective variety, 222
 Blow-up, of an ideal sheaf on a quasi-projective variety, 223
 Blow-up, of an ideal sheaf on an affine variety, 221
 Buchsbaum, 25

 Canonical divisor, 286
 Cartier divisor, 245, 273, 291
 Carvajal-Rojas, 433
 Cascini, 387
 Castelnuovo, 328, 383
 Castelnuovo's contraction theorem, 383
 Chain map, 307
 Chevalier, 411
 Chinese remainder theorem, 4
 Christiansen, 235
 Clifford, 342
 Clifford's theorem, 342
 Closed embedding, of a quasi-affine variety, 55
 Closed embedding, of a quasi-projective variety, 88
 Closed embedding, of an affine algebraic set, 38
 Closed subscheme, 289
 Codimension, of a quasi-affine algebraic set, 45
 Codimension, of a quasi-projective algebraic set, 141
 Cohen, 410
 Coherent \mathcal{O}_X -module, 200
 Coherent sheaf, 290

 Cohomology, 311, 312
 Complete abstract variety, 295
 Complete linear system, 260
 Complex, 307
 Complex manifold, 175
 Constant sheaf, 189
 Coordinate functions, on an affine algebraic set, 38
 Coordinate ring of \mathbb{P}^n , 68
 Coordinate ring, of a biprojective variety, 103
 Coordinate ring, of a projective algebraic set, 70
 Coordinate ring, of a subvariety of $\mathbb{A}^m \times \mathbb{P}^n$, 105
 Cossart, 228
 Curve, 45
 Cutkosky, 235, 236, 345

 Dedekind, xi
 Degree, of a divisor on a curve, 254
 Degree, of a graded module, 301
 Degree, of a map of curves, 254
 Degree, of a projective subscheme, 301
 Degree, of a regular map, 371, 406
 Degree, of an invertible sheaf on a curve, 259
 Del Pezzo, 304
 Derivation, 279
 Diagonal, 106, 176
 Dimension, of a quasi-projective algebraic set, 139
 Dimension, of a topological space, 42
 Direct limit, 181
 Discrete valuation, 395
 Discrete valuation ring, 395
 Discriminant ideal, 398
 Divisor, 240
 Divisor class group, 242
 Divisor of a function, 241
 Divisor of a section of an invertible sheaf, 273
 Divisor of poles of a function, 242
 Divisor of zeros of a function, 241
 Divisor, local equation, 245
 Divisor, of a form, 249
 Divisor, of a rational differential n -form, 286
 Divisor, support, 240
 Divisorial valuation, 229
 Dominant map, of affine algebraic sets, 37

- Dominant rational map, of affine varieties, 59
 Dominant rational map, of quasi-projective varieties, 94
 Dominant regular map, of a quasi-projective variety, 82
 Dominate, rings, 1
 dvr, 395

 Effective Cartier divisor, 291
 Effective divisor, 240
 Eisenbud, 304, 330
 Elliptic curve, 341
 Étale regular map, 426
 Euclidean topology, 175
 Euler, 13
 Euler characteristic of a sheaf, 319
 Excision, 331

 Fiber cone, 294
 Finite map, 127
 Finite map, of affine varieties, 40
 First theorem of Bertini, 459
 Fractional ideal, 243
 Frobenius homomorphism, 349

 Gabrièlov, 420
 Galois over X , 422
 Galois regular map, 422
 Generated by global sections, 212
 Genus, of a curve, 334
 Geometric regularity theorem, 328
 Geometrically integral ring, 452
 Geometrically integral scheme, 452
 Geometrically irreducible ring, 452
 Geometrically irreducible scheme, 452
 Geometrically reduced ring, 452
 Geometrically reduced scheme, 452
 Germ, 185
 Godement, 181
 Going down theorem, 393
 Going up theorem, 393
 Goto, 324, 330
 Graded module, 63
 Graded ring, 63
 Grant, 433
 Graph, of a rational map, 107
 Graph, of a regular map, 106
 Grauert, 387
 Grothendieck, xii, 151, 367, 433

 Hacon, 387
 Harris, 304

 Hartshorne, xii, 181
 Heinzer, 395
 Herzog, 46, 116
 Higher direct images of a sheaf, 320
 Hilbert, 11, 29, 301, 376
 Hilbert polynomial, of a graded module, 301
 Hilbert polynomial, of a projective subscheme, 301
 Hilbert's basis theorem, 11
 Hilbert's nullstellensatz, 29
 Hilbert-Samuel polynomial, 376
 Hironaka, 227, 232, 234, 295
 Hodge index theorem, 382
 Homogeneous coordinates, 90
 Homogeneous coordinates, on \mathbb{P}^n , 68
 Homogeneous ideal, 64
 Homogeneous ideal of a subscheme, 292
 Homotopy, 308
 Huneke, 116
 Hurwitz, 348

 Ideal sheaf, 120, 205
 Ideal sheaf, of a subvariety, 200
 Ideal sheaf, on a projective variety, 192
 Ideal sheaf, on an affine variety, 192
 Ideal transform, 56
 Image, of a rational map, 94
 Indirect limit, 184
 Inertia field, 440
 Injective sheaf homomorphism, 194
 Integral scheme, 290
 Intersection number, 368
 Invertible sheaf, 203, 223, 245, 269, 290
 Irreducible scheme, 290
 Irreducible topological space, 29
 Isomorphism, of affine algebraic sets, 38
 Isomorphism, of quasi-affine varieties, 54
 Isomorphism, of varieties, 76

 Jacobi, 360
 Jacobian criterion for nonsingularity, 160
 Jacobian, of a curve, 360, 361
 Jung, 435

 Kähler differentials, 280
 Kaplansky, 232
 Karu, 234, 236
 Kollár, 387
 Krull, 20

- Krull's principal ideal theorem, 20
 Künneth, 319
 Künneth formula, 319

 Lang, 29
 Lazarsfeld, 345
 Linear equivalence of divisors, 242
 Linear subspace, of a projective space, 95
 Linear system, 260
 Lipman, 227, 231
 Local cohomology, 326, 330
 Local equations, 155
 Local ring, 1
 Locally free sheaf, 217
 Locally principal ideal sheaf, 223
 Locally ringed space, 191

 Map of complexes, 307
 Matsuki, 234
 Matsusaka, 452
 Mayer-Vietoris sequence, 331
 McKernan, 387
 Milne, 362
 Mori, 387
 Morphism, of affine schemes, 297
 Morphism, of locally ringed spaces, 191
 Morphism, of schemes, 297
 Multiplicity, 376
 Mumford, xii, 328, 330
 Mustață, 345

 Nagata, 57, 324, 433
 Nakayama, 8
 Newton, 437
 Nishida, 324
 Noether, 17, 134
 Noether's normalization lemma, 17
 Nondegenerate variety, 299
 Nonsingular point, of a variety, 160
 Nonsingular, quasi-projective variety, 160
 Normal field extension, 394
 Normal point, 135
 Normal variety, 135
 Normalization, in a finite extension, 135
 Nullhomotopic, 308
 Numerical equivalence of divisors, 382
 Numerical polynomial, 300

 Oda, 235
 Open embedding, of quasi-affine varieties, 55

 Open subscheme, 289

 Picard group, 270
 Piltant, 228
 Presheaf, 185
 Presheaf, homomorphism, 185
 Presheaf, isomorphism, 185
 Prime divisor, 240
 Primitive element, 401
 Primitive element theorem, 7
 Principal divisor, 242
 Projective algebraic set, 69
 Projective Noether normalization, 134
 Projective scheme, 297
 Projective subscheme, 289
 Projective variety, 69
 Proper transform, 123
 Purity of the branch locus, 433

 Quadratic transform, 170
 Quasi-affine variety, 34
 Quasi-coherent \mathcal{O}_X -module, 200
 Quasi-coherent sheaf, 290
 Quasi-projective algebraic set, 69
 Quasi-projective subscheme, 289
 Quasi-projective variety, 69
 Quotient of Y by H , 422

 Ramification index, of a regular map of curves, 346
 Ramification locus, in the domain of a finite map, 432, 433
 Ramification locus, in the range of a finite map, 407
 Ramified extension, of a normal domain, 400
 Ramified extension, of local domains, 399
 Ramified, map of curves, 346
 Rashid, 234
 Rational function field, 7
 Rational functions, on a biprojective variety, 103
 Rational functions, on a projective variety, 74
 Rational functions, on a quasi-affine variety, 53
 Rational functions, on a quasi-projective variety, 82
 Rational functions, on an affine variety, 49
 Rational map, of affine varieties, 58

- Rational map, of projective varieties, 93
 Rational map, of quasi-projective varieties, 93
 Rational maps, on a subvariety of $\mathbb{A}^m \times \mathbb{P}^n$, 105
 Reduced ramification index, 396
 Reduced scheme, 290
 Rees, 57
 Reflexive rank 1 sheaf, 245
 Regular functions, on \mathbb{A}^n , 27
 Regular functions, on a quasi-affine variety, 49
 Regular functions, on an affine algebraic set, 33
 Regular functions, on an open subset of a biprojective variety, 103
 Regular map, of affine algebraic sets, 36
 Regular map, of an affine variety to \mathbb{A}^n , 35
 Regular map, of quasi-affine varieties, 54
 Regular map, of varieties, 76
 Regular maps, of biprojective varieties, 103
 Regular parameters, 154
 Regularity, 328
 Relative degree, 396
 Resolution of indeterminacy, 225
 Resolution of singularities, 225
 Riemann, 335, 340
 Riemann-Roch inequality, 335
 Riemann-Roch problem, 344
 Riemann-Roch theorem, 340
 Rigidity lemma, 358
 Roberts, 324
 Roch, 335, 340

 Sally, 46, 232
 Samuel, 376
 Scheme, 289, 297
 Scheme-theoretic fiber, 290
 Scheme-theoretic intersection, 290
 Schwede, 433
 Second theorem of Bertini, 458
 Section, nonzero divisor, 291
 Separable field extension, 452
 Separable regular map, 346, 452
 Separably generated field, 7
 Separated scheme, 297
 Separated, variety, 82
 Separating transcendence basis, 7
 Serre, xii, 301, 319, 335, 339, 468
 Serre duality, 319, 339
 Shafarevich, xii, 168
 Shannon, 232
 Sheaf, 186
 Sheaf cohomology, 308
 Sheaf, axioms, 186
 Sheaf, global section, 186
 Sheaf, homomorphism, 187
 Sheaf, isomorphism, 187
 Sheaf, support, 203
 Sheafification of a graded module on a projective variety, 199
 Sheafification of a module on an affine variety, 196
 Sheafification of a presheaf, 187
 Short exact sequence of sheaves, 194
 Simis, 116
 Singular cohomology, 358
 Singular locus, 162
 Smooth morphism, 456
 Snapper, 366
 Snapper polynomial, 366
 Splitting field, 438
 Splitting group, 438
 Srinivas, 345
 Strict transform, of a projective variety, 123
 Strict transform, of an affine variety, 116
 Subscheme, 289
 Subsheaf, 192
 Subvariety, 69
 Surface, 45
 Surjective sheaf homomorphism, 194
 Symbolic power, 245
 System of parameters, 142

 Tamely ramified, map of curves, 346
 Tangent cone, 294
 Tangent space, extrinsic definition, 156
 Tangent space, intrinsic definition, 157
 Torsion sheaf, 366
 Transcendence basis, 6
 Transcendence degree, 6
 Transition functions on an invertible sheaf, 271
 Trivialization, of an invertible sheaf, 292
 Tucker, 433

 Ulrich, 116
 Uniformizing parameters, 285
 Universal property of blowing up, 223

- Unramified extension, of a normal domain, 400
- Unramified extension, of local domains, 399

- Valuation, 228
- Valuation ring, 228
- Value group, 228
- Variety, biprojective, 103
- Variety, quasi-biprojective, 103
- Vasconcelos, 116
- Veronese subring, 64
- Very ample divisor, 262
- Very ample invertible sheaf, 270

- Walker, 227
- Watanabe, 324
- Weber, xi
- Weierstrass, 176
- Weierstrass preparation theorem, 176
- Weil, 362
- Wildly ramified, map of curves, 346
- Włodarczyk, 234

- Zariski, xii, 76, 148, 151, 161, 168, 227, 229–232, 344, 388, 392, 413, 416, 420, 433, 452, 458
- Zariski topology, on \mathbb{A}^n , 29
- Zariski topology, on \mathbb{P}^n , 68
- Zariski topology, on a quasi-projective algebraic set, 68
- Zariski topology, on an affine algebraic set, 34
- Zariski topology, on the Proj of a graded ring, 67
- Zariski topology, on the Spec of a ring, 5
- Zariski's connectedness theorem, 151
- Zariski's main theorem, 148, 416
- Zariski's subspace theorem, 420