Contents

Preface to the second edition xi

Preface to the first edition xiii

Chapter 1. Introduction 1

§1.1. Partial Differential Equations 1

§1.2. Examples 3

§1.3. Strategies for Studying PDE 6

§1.4. Overview 8

§1.5. Problems 11

§1.6. References 12

Part I. REPRESENTATION FORMULAS FOR SOLUTIONS

Chapter 2. Four Important Linear Partial Differential Equations 17

§2.1. Transport Equation 17

§2.2. Laplace's Equation 19

§2.3. Heat Equation 42

§2.4. Wave Equation 63

§2.5. Problems 81

§2.6. References 86

Chapter 3. Nonlinear First-Order PDE 89

§3.1. Complete Integrals, Envelopes 90

§3.2. Characteristics 94

§3.3. Introduction to Hamilton–Jacobi Equations 110
§3.4. Introduction to Conservation Laws 130
§3.5. Problems 154
§3.6. References 158

Chapter 4. Other Ways to Represent Solutions 159
§4.1. Separation of Variables 159
§4.2. Similarity Solutions 167
§4.3. Transform Methods 177
§4.4. Converting Nonlinear into Linear PDE 195
§4.5. Asymptotics 200
§4.6. Power Series 220
§4.7. Problems 231
§4.8. References 236

Part II. THEORY FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS

Chapter 5. Sobolev Spaces 239
§5.1. Hölder Spaces 240
§5.2. Sobolev Spaces 241
§5.3. Approximation 249
§5.4. Extensions 253
§5.5. Traces 256
§5.6. Sobolev Inequalities 260
§5.7. Compactness 270
§5.8. Additional Topics 273
§5.9. Other Spaces of Functions 282
§5.10. Problems 289
§5.11. References 292

Chapter 6. Second-Order Elliptic Equations 293
§6.1. Definitions 293
§6.2. Existence of Weak Solutions 297
§6.3. Regularity 307
§6.4. Maximum Principles 323
§6.5. Eigenvalues and Eigenfunctions 333

§6.6. Problems 343
§6.7. References 348

Chapter 7. Linear Evolution Equations 349
§7.1. Second-Order Parabolic Equations 349
§7.2. Second-Order Hyperbolic Equations 375
§7.3. Systems of Hyperbolic First-Order Equations 396
§7.4. Semigroup Theory 409
§7.5. Problems 420
§7.6. References 423

Part III. THEORY FOR NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Chapter 8. The Calculus of Variations 427
§8.1. Introduction 427
§8.2. Existence of Minimizers 437
§8.3. Regularity 454
§8.4. Constraints 458
§8.5. Critical Points 471
§8.6. Invariance, Noether's Theorem 481
§8.7. Problems 489
§8.8. References 494

Chapter 9. Nonvariational Techniques 495
§9.1. Monotonicity Methods 495
§9.2. Fixed Point Methods 501
§9.3. Method of Subsolutions and Supersolutions 510
§9.4. Nonexistence of Solutions 514
§9.5. Geometric Properties of Solutions 521
§9.6. Gradient Flows 526
§9.7. Problems 538
§9.8. References 541

Chapter 10. Hamilton–Jacobi Equations 543
§10.1. Introduction, Viscosity Solutions 543
§10.2. Uniqueness 550

§10.3. Control Theory, Dynamic Programming 554
§10.4. Problems 566
§10.5. References 569

Chapter 11. Systems of Conservation Laws 571
§11.1. Introduction 571
§11.2. Riemann's Problem 582
§11.3. Systems of Two Conservation Laws 595
§11.4. Entropy Criteria 601
§11.5. Problems 613
§11.6. References 616

Chapter 12. Nonlinear Wave Equations 617
§12.1. Introduction 617
§12.2. Existence of Solutions 620
§12.3. Semilinear Wave Equations 627
§12.4. Critical Power Nonlinearity 636
§12.5. Nonexistence of Solutions 642
§12.6. Problems 647
§12.7. References 651

Appendices 653
§A. Notation 653
§B. Inequalities 661
§C. Calculus 665
§D. Functional Analysis 674
§E. Measure Theory 684

Bibliography 689

Index 703