
Contents

Preface	vii
Chapter 1. Introduction and Preliminary Material	1
§1.1. A simple example	1
§1.2. A little motivation and history	7
§1.3. Notation	7
§1.4. Maximum principle and barrier functions	8
§1.5. Asymptotic expansions	10
Chapter 2. Convection-Diffusion Problems in One Dimension	15
§2.1. Asymptotic analysis—an extended example	15
§2.2. Green’s functions	21
§2.3. A priori bounds on the solution and its derivatives	24
§2.4. Decompositions of the solution	38
Chapter 3. Finite Difference Methods in One Dimension	43
§3.1. M-matrices, upwinding	45
§3.2. Artificial diffusion	54
§3.3. Uniformly convergent schemes	56
§3.4. Shishkin meshes	59
Chapter 4. Convection-Diffusion Problems in Two Dimensions	69
§4.1. General description	69
§4.2. A priori estimates	77
§4.3. General comments on numerical methods	84

Chapter 5. Finite Difference Methods in Two Dimensions	87
§5.1. Extending one-dimensional approaches	87
§5.2. Shishkin meshes	89
§5.3. Characteristic boundary layers	91
§5.4. Other remarks	93
Chapter 6. Finite Element Methods	95
§6.1. The loss of stability in the (Bubnov-)Galerkin FEM	95
§6.2. Relationship to classical FEM analysis	98
§6.3. L^* -splines	100
§6.4. The streamline-diffusion finite element method (SUPG)	103
§6.5. Stability of the Galerkin FEM for higher-degree polynomials	111
§6.6. Shishkin meshes	117
§6.7. Discontinuous Galerkin finite element method	126
§6.8. Continuous interior penalty (CIP) method	131
§6.9. Adaptive methods	139
Chapter 7. Concluding Remarks	143
Bibliography	145
Index	155