
Preface

This book is written for students and researchers in applied mathematics with an interest in science and engineering. Our main purpose is to provide a mathematically solid introduction to the basic ideas and tools in probability theory and stochastic analysis. Starting from the basics of random variables and probability theory, we go on to discuss limit theorems, Markov chains, diffusion processes, and random fields. Since the kind of readers we have in mind typically have some background in differential equations, we put more weight on the differential equation approach. In comparison, we have neglected entirely martingale theory even though it is a very important part of stochastic analysis. The diffusion process occupies a central role in this book. We have presented three different ways of looking at the diffusion process: the approach of using stochastic differential equations, the Fokker-Planck equation approach, and the path integral approach. The first allows us to introduce stochastic calculus. The second approach provides a link between differential equations and stochastic analysis. The path integral approach is very much preferred by physicists and is also suited for performing asymptotic analysis. In addition, it can be extended to random fields.

In choosing the style of the presentation, we have tried to strike a balance between rigor and the heuristic approach. We have tried to give the reader an idea about the kind of mathematical construction or mathematical argument that goes into the subject matter, but at the same time, we often stop short of proving all the theorems we state or we prove the theorems under stronger assumptions. Whenever possible, we have tried to give the intuitive picture behind the mathematical constructions.

Another emphasis is on numerical algorithms, including Monte Carlo methods, numerical schemes for solving stochastic differential equations, and the stochastic simulation algorithm. The book ends with a discussion on two application areas, statistical mechanics and chemical kinetics, and a discussion on rare events, which is perhaps the most important manifestation of the effect of noise.

The material contained in this book has been taught in various forms at Peking University, Princeton University, and New York University since 2001. It is now a required course for the special applied mathematics program at Peking University.

Weinan E
Tiejun Li
Eric Vanden-Eijnden

December 2018