

---

# Contents

Preface	vii
Some basic notation	xiii
Chapter 1. Basic calculus in the complex domain	1
§1.1. Complex numbers, power series, and exponentials	3
Exercises	12
§1.2. Holomorphic functions, derivatives, and path integrals	14
Exercises	24
§1.3. Holomorphic functions defined by power series	26
Exercises	32
§1.4. Exponential and trigonometric functions: Euler's formula	34
Exercises	41
§1.5. Square roots, logs, and other inverse functions	44
Exercises	48
§1.6. Pi is irrational	58
Chapter 2. Going deeper – the Cauchy integral theorem and consequences	61
§2.1. The Cauchy integral theorem and the Cauchy integral formula	63
Exercises	71
§2.2. The maximum principle, Liouville's theorem, and the fundamental theorem of algebra	73
Exercises	76

---

§2.3. Harmonic functions on planar domains	78
Exercises	87
§2.4. Morera's theorem, the Schwarz reflection principle, and Goursat's theorem	89
Exercises	92
§2.5. Infinite products	93
Exercises	106
§2.6. Uniqueness and analytic continuation	107
Exercises	112
§2.7. Singularities	114
Exercises	117
§2.8. Laurent series	118
Exercises	122
§2.9. Green's theorem	123
§2.10. The fundamental theorem of algebra (elementary proof)	128
§2.11. Absolutely convergent series	130
Chapter 3. Fourier analysis and complex function theory	135
§3.1. Fourier series and the Poisson integral	137
Exercises	150
§3.2. Fourier transforms	152
Exercises	159
More general sufficient condition for $f \in \mathcal{A}(\mathbb{R})$	160
Fourier uniqueness	162
§3.3. Laplace transforms and Mellin transforms	163
Exercises	165
The matrix Laplace transform and Duhamel's formula	167
§3.4. Inner product spaces	169
§3.5. The matrix exponential	172
§3.6. The Weierstrass and Runge approximation theorems	174
Chapter 4. Residue calculus, the argument principle, and two very special functions	183
§4.1. Residue calculus	186
Exercises	193
§4.2. The argument principle	196
Exercises	201

---

§4.3. The Gamma function	203
Exercises	207
The Legendre duplication formula	209
§4.4. The Riemann zeta function and the prime number theorem	211
Counting primes	219
The prime number theorem	221
Exercises	225
§4.5. Euler's constant	227
§4.6. Hadamard's factorization theorem	233
Chapter 5. Conformal maps and geometrical aspects of complex function theory	243
§5.1. Conformal maps	246
Exercises	254
§5.2. Normal families	255
Exercises	256
§5.3. The Riemann sphere and other Riemann surfaces	257
Exercises	265
§5.4. The Riemann mapping theorem	268
Exercises	271
§5.5. Boundary behavior of conformal maps	272
Exercises	275
§5.6. Covering maps	277
Exercises	279
§5.7. The disk covers the twice-punctured plane	280
Exercises	281
§5.8. Montel's theorem	284
Exercises on Fatou sets and Julia sets	286
§5.9. Picard's theorem	289
Exercises	290
§5.10. Harmonic functions II	290
Exercises	302
§5.11. Surfaces and metric tensors	303
§5.12. Poincaré metrics	311
§5.13. Groups	318

---

Chapter 6. Elliptic functions and elliptic integrals	323
§6.1. Periodic and doubly periodic functions	325
Exercises	328
§6.2. The Weierstrass P-function in elliptic function theory	331
Exercises	334
§6.3. Theta functions and elliptic functions	337
Exercises	341
§6.4. Elliptic integrals	342
Exercises	348
§6.5. The Riemann surface of the square root of a cubic	350
Exercises	355
§6.6. Rapid evaluation of the Weierstrass P-function	358
Rectangular lattices	359
Chapter 7. Complex analysis and differential equations	363
§7.1. Bessel functions	366
Exercises	381
§7.2. Differential equations on a complex domain	382
Exercises	403
§7.3. Holomorphic families of differential equations	405
Exercises	410
§7.4. From wave equations to Bessel and Legendre equations	412
Appendix A. Complementary material	417
§A.1. Metric spaces, convergence, and compactness	418
Exercises	430
§A.2. Derivatives and diffeomorphisms	431
§A.3. The Laplace asymptotic method and Stirling's formula	437
§A.4. The Stieltjes integral	441
§A.5. Abelian theorems and Tauberian theorems	448
§A.6. Cubics, quartics, and quintics	459
Bibliography	473
Index	477